Higher – Unit 2- Factorisation

An alternative method

Factorising

Factorising will play a big part in your study of unit 2.

There are 3 types of factor important to us:

Common Factor $\longrightarrow$ $16xy^2 + 8y$

$= 8y(2xy + 1)$

D O T S $\longrightarrow$ $9r^2 - 25q^2$

$= (3r + 5q)(3r - 5q)$

Tri-nomials $\longrightarrow$ $2x^2 - 7x + 6$

$= (2x - 3)(x - 2)$

It is the last of these that we are going to practice.

The Quadratic Formula

You will recall from previous work that the quadratic formula can be used to find the roots of almost any quadratic:

for $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Generally, this is used when the quadratic cannot be factorised as factorisation is deemed quicker otherwise. However if you have problems factorising the following could help you factorise quadratics where possible.

Example 1

Factorise $3y^2 + 8y + 4$

Solution:

a = 3, b = 8, c = 4

$$y = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$b^2 - 4ac = 8^2 - 4(3)(4)$$
$$= 64 - 48$$
$$= 16$$

Should be perfect square →

$$y = \frac{-8 \pm \sqrt{16}}{2(3)}$$

$$y = -\cancel{\frac{4}{6}} = -\frac{2}{3}$$ **Or** $$y = -2$$

$$3y = -2$$

$$\boxed{3y + 2} = 0$$

$$\boxed{y + 2} = 0$$

So $3y^2 + 8y + 4 = (3y + 2)(y + 2)$

Example 2

Factorise $2x^2 + 9x + 4$

Solution:

a = 2, b = 9, c = 4

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$b^2 - 4ac = 9^2 - 4(2)(4)$$

$$= 81 - 32$$

Should be perfect square → $= 49$

$$x = \frac{-9 \pm \sqrt{49}}{2(2)}$$

$$x = \cancel{-\frac{2}{4}} = -\frac{1}{2}$$

$$2x = -1$$

$$\boxed{2x+1} = 0$$

Or

$$x = -4$$

$$\boxed{x+4} = 0$$

So $2x^2 + 9x + 4 \quad = (2x+1)(x+4)$

Example 3

Factorise $6m^2 + 13m + 5$

Solution:

a = 6, b = 13, c = 5

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$b^2 - 4ac = 13^2 - 4(6)(5)$$

$$= 169 - 120$$

Should be perfect square → $= 49$

$$x = \frac{-13 \pm \sqrt{49}}{2(6)}$$

$$x = -\cancel{\frac{6}{12}} = -\frac{1}{2}$$ **Or** $$x = -\cancel{\frac{20}{12}} = -\frac{5}{3}$$

$$2x = -1 \qquad\qquad 3x = -5$$

$$\boxed{2x+1} = 0 \qquad\qquad \boxed{3x+5} = 0$$

So $6m^2 + 13m + 5 = (2m+1)(3m+5)$

Example 4

Factorise $10d^2 - 11d - 6$

Watch double negative

Solution:

a = 10, b = -11, c = -6

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$b^2 - 4ac = (-11)^2 - 4(10)(-6)$$

$$= 121 + 240$$

Should be perfect square $\longrightarrow$

$$= 361$$

$$x = \frac{11 \pm \sqrt{361}}{2(10)}$$

$$x = \cancel{\frac{30}{20}} = \frac{3}{2}$$

Or

$$x = -\cancel{\frac{8}{20}} = -\frac{2}{5}$$

$$2x = 3$$

$$5x = -2$$

$$\boxed{2x - 3} = 0$$

$$\boxed{5x + 2} = 0$$

So $10d^2 - 11d - 6 = (2d - 3)(5d + 2)$

Example 5

Factorise $9p^2+18p-16$

Watch double negative

Solution:

a = 9, b = 18, c = -16

$$x=\frac{-b\pm\sqrt{(b^2-4ac)}}{2a}$$

$$b^2-4ac=(18)^2-4(9)(-16)$$
$$=324+576$$

Should be perfect square $\rightarrow$ $=900$

$$x=\frac{-18\pm\sqrt{900}}{2(9)}$$

$x=\cancel{\frac{12}{18}}=\frac{2}{3}$ **Or** $x=-\cancel{\frac{48}{18}}=-\frac{8}{3}$

$3x=2$ $\quad$ $3x=-8$

$\boxed{3x-2}=0$ $\quad$ $\boxed{3x+8}=0$

So $9p^2+18p-16=(3p-2)(3p+8)$