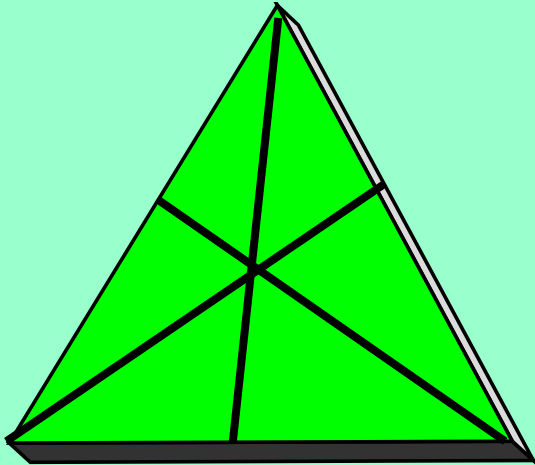


9.

Concurrence

3 or more lines meeting at a point

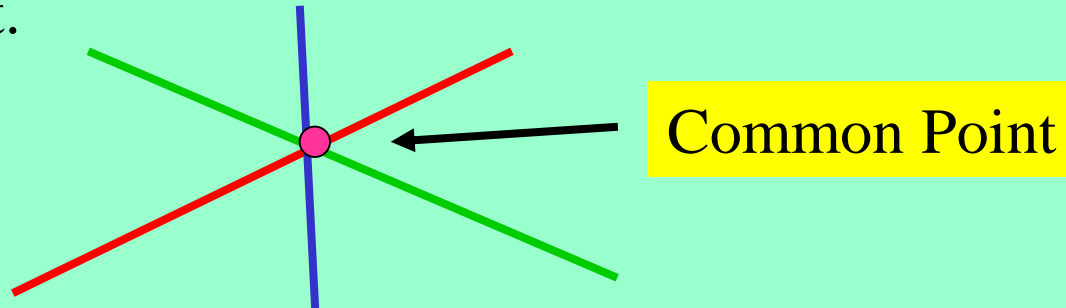


Concurrence

Copy the following:

Intersection deals with two lines meeting at a common point.

Concurrence deals with three or more lines passing through a common point.



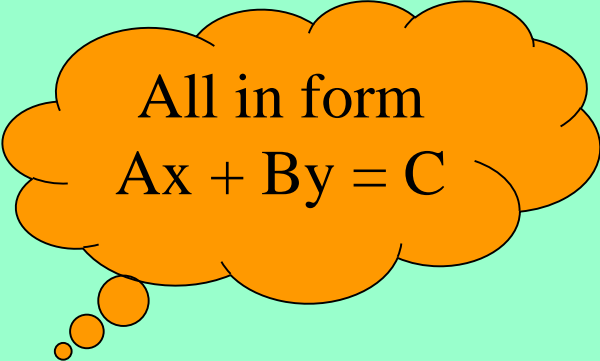
To prove that lines are concurrent:

1. Take the equations of two of the lines to set up and solve a simultaneous equation.
2. Prove that the solution to part 1 also lies on the other line(s).

Example 1

Show that the following lines are concurrent:

$$x + y - 9 = 0 \quad 2x + 3y = 23 \quad 3x - y = 7$$



All in form
 $Ax + By = C$

Solution :

1. Take one pair of equations and solve to find point of intersection

$$\left. \begin{array}{l} x + y = 9 \\ 2x + 3y = 23 \end{array} \right\}$$

Point of Intersection $\longrightarrow (4, 5)$

Example 1

Show that the following lines are concurrent:

$$\cancel{x + y - 9 = 0} \quad \cancel{2x + 3y = 23} \quad 3x - y = 7$$

Solution :

2. Show that the point you have just found also lies on the other line(s)

3. Make a statement

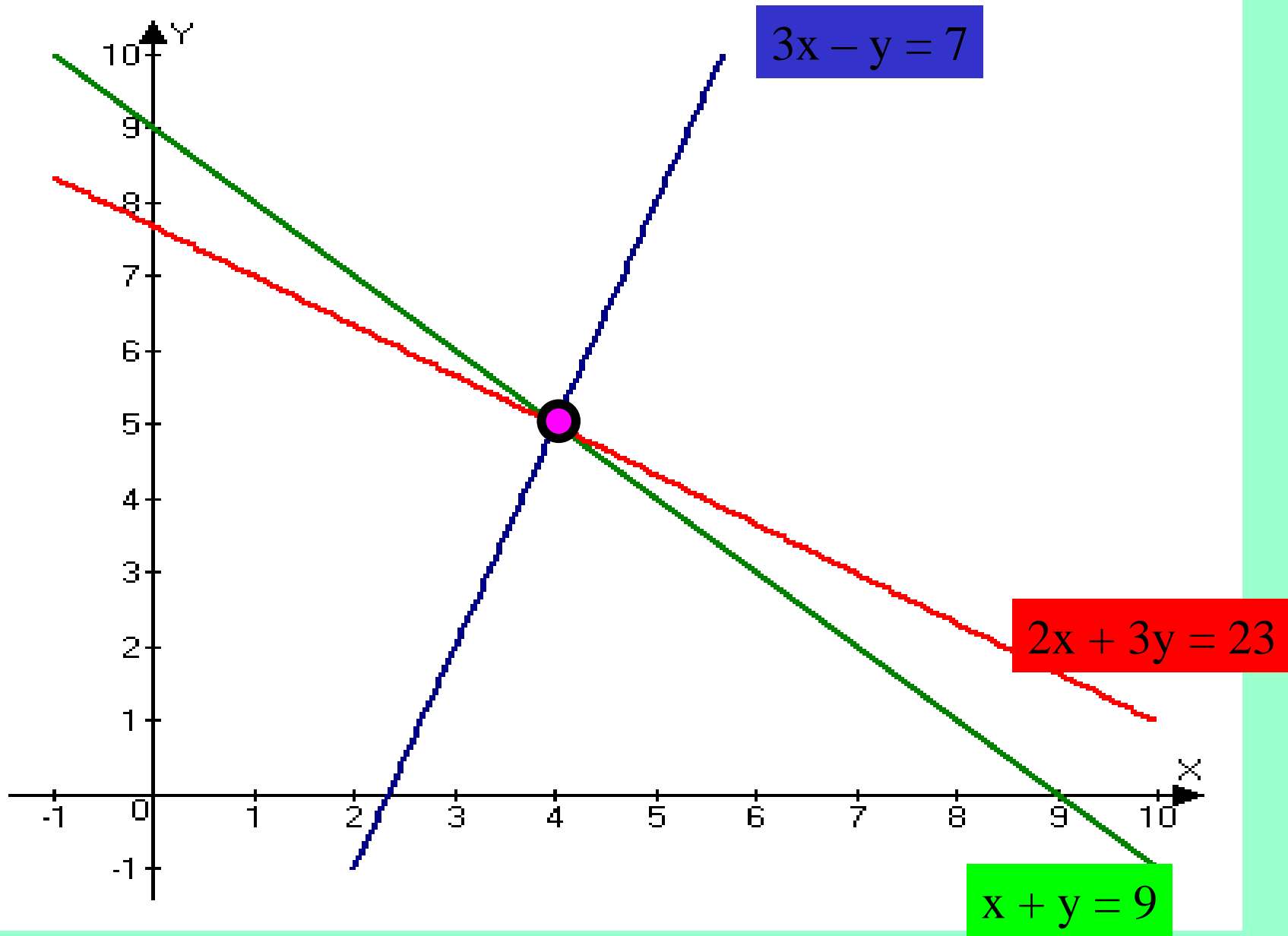
Point of Intersection $\longrightarrow (4, 5)$

Sub $x = 4$ & $y = 5$ in :

$$\begin{aligned} \text{LHS: } & 3x - y \\ & = 3(4) - 5 \\ & = 7 \end{aligned}$$

$$\text{RHS: } = 7$$

AS LHS=RHS lines are concurrent



Example 2

Find k if these lines are concurrent:

$$x + 4y = 7 \quad 3x + y = 10 \quad x - 5y + k = 0$$

All in form
 $Ax + By = C$

Solution :

1. Take one pair of equations and solve to find point of intersection

$$\left. \begin{array}{l} x + 4y = 7 \\ 3x + y = 10 \end{array} \right\} \begin{array}{l} \times 3 \\ \times 1 \end{array} \left\{ \begin{array}{l} 3x + 12y = 21 \\ \underline{3x + y = 10} \end{array} \right\}$$

$$11y = 11$$

$$y = 1$$

Sub $y = 1$ into $x + 4y = 7$

$$x + 4(1) = 7$$

$$x = 3$$

Point of
Intersection $\longrightarrow (3, 1)$

Example 2

Find k if these lines are concurrent:

$$~~x + 4y = 7~~$$

$$~~3x + y = 10~~$$

$$x - 5y + k = 0$$

Solution :

2. Points ARE concurrent so substitute into 3rd equation to find k .

Point of Intersection $\longrightarrow (3, 1)$

Sub $x = 3$ & $y = 1$ in :

$$x - 5y + k = 0$$

$$\Rightarrow 3 - 5(1) + k = 0$$

$$\Rightarrow -2 + k = 0$$

$$k = 2$$