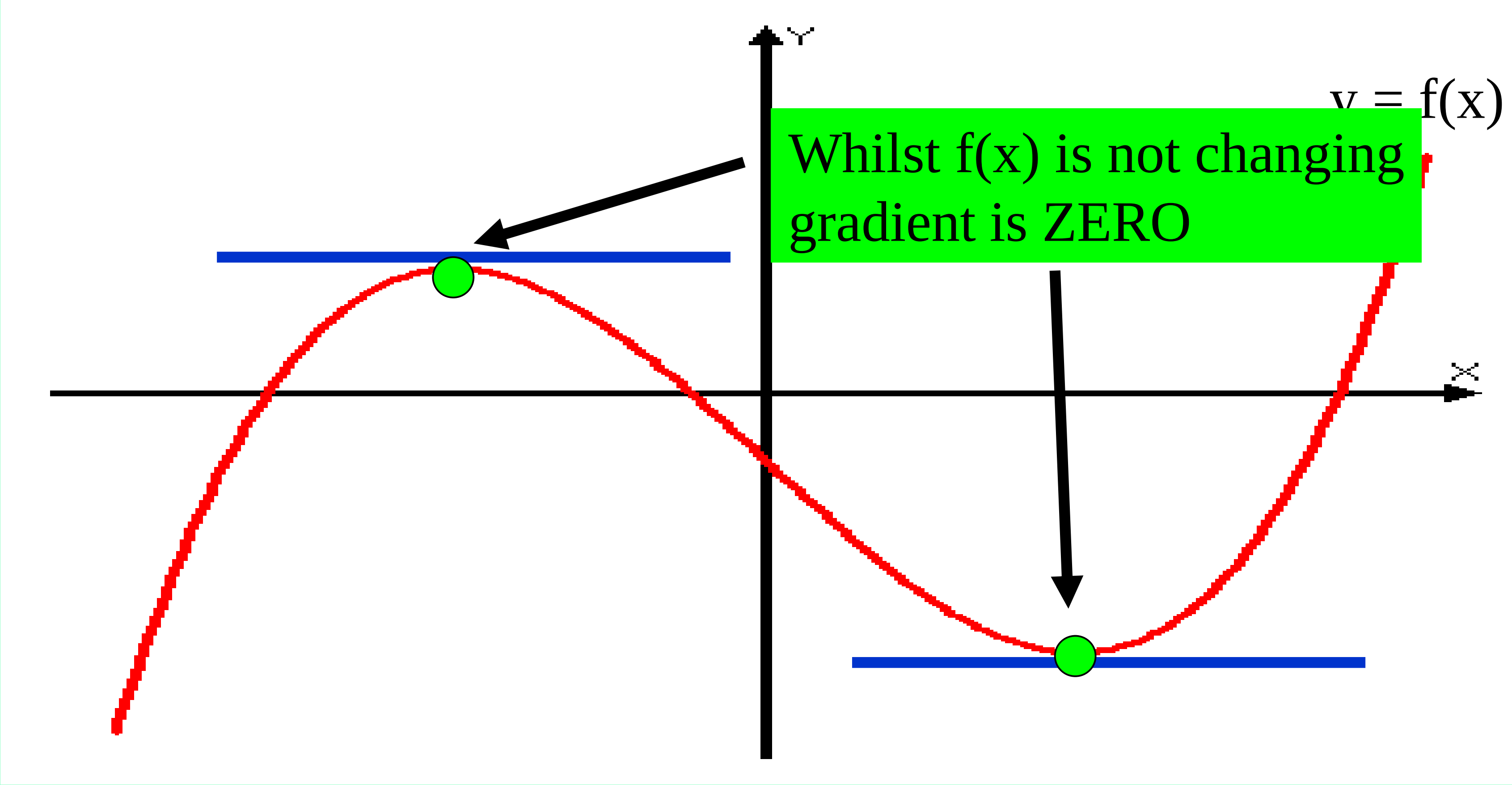
8.

Stationary Points

$$f'(x) = 0$$

What if the function is neither increasing nor decreasing?



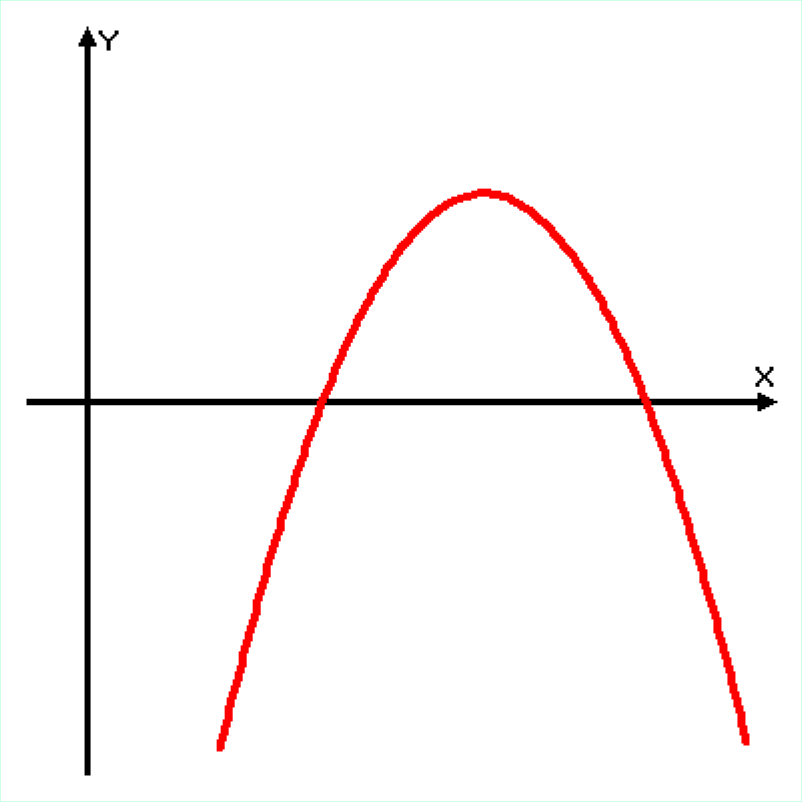
Recall from the last lesson that when the gradient is changing there will be points where the tangent is horizontal and so the gradient=ZERO

These are called **stationary points.** At these points $f'(x) = 0$

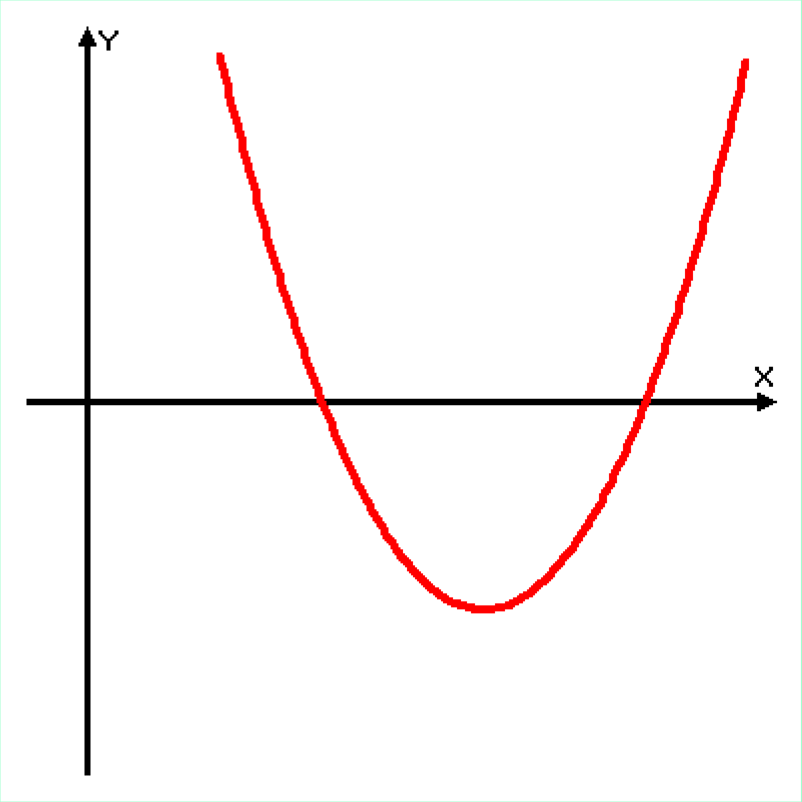
The Nature of a Stationary Point

Copy the following:

The nature of a stationary point is determined by the gradient either side of it. There are four classifications:



Maximum Turning Point

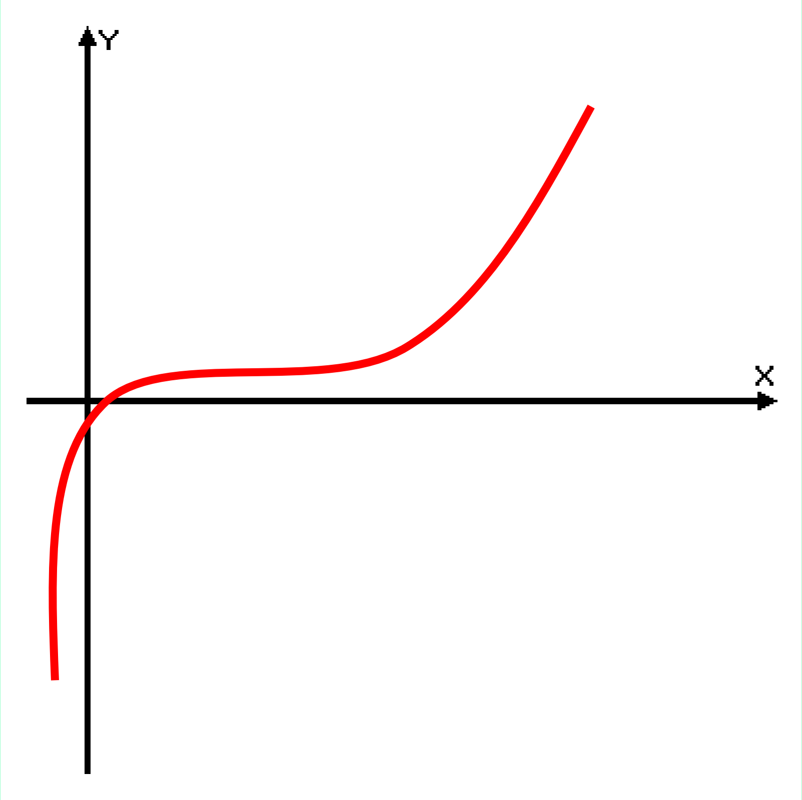


Minimum Turning Point

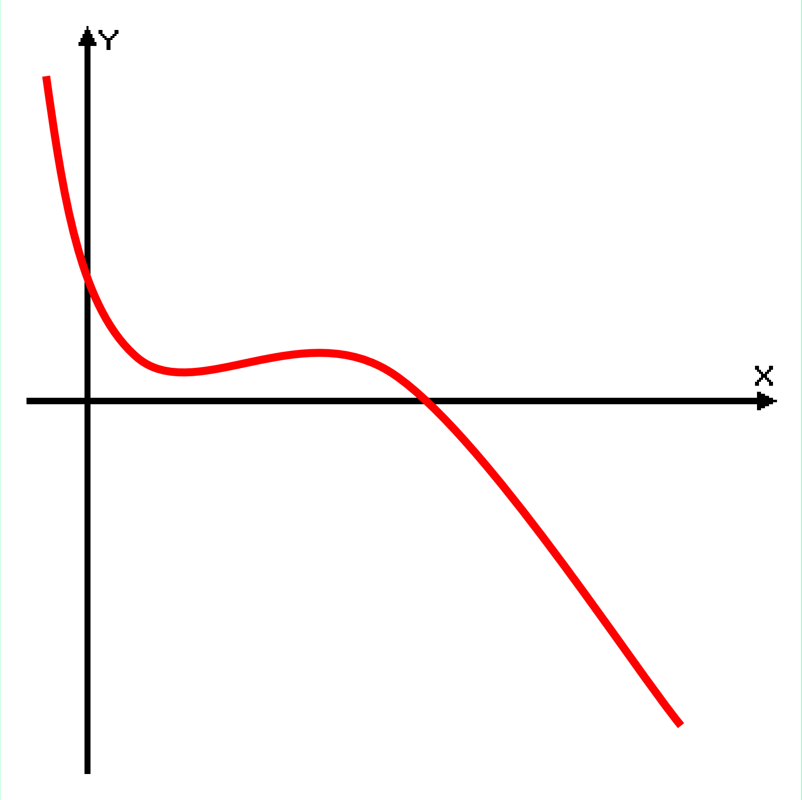
The Nature of a Stationary Point

Copy the following:

The nature of a stationary point is determined by the gradient either side of it. There are four classifications:



Rising Point of Inflection



Falling Point of Inflection

Example 1

NAB

Determine the coordinates and nature of the stationary points on the curve $y = 5x^3(x-4)$

Solution:

1. Prepare for differentiation

2. Find dy / dx

3. Set dy/dx equal to zero, factorise if possible, and solve for x

$$y = 5x^3(x-4)$$

$$y = 5x^4 - 20x^3$$

$$\frac{dy}{dx} = 20x^3 - 60x^2$$

At SP's dy/dx = 0

In exams must make this statement

$$0 = 20x^3 - 60x^2$$

$$0 = 20x^2(x-3)$$

$20x^2 = 0$ or $(x-3) = 0$

x = 0 or x = 3

Example 1

Determine the coordinates and nature of the stationary points on the curve $y = 5x^3(x-4)$

Solution:

4. Find y-coordinates by subbing these values into original equation **(NOT dy/dx)**

5. State coords of Stationary Points

x = 0 or x = 3

For x = 0:

$y = 5(0)^3((0)-4)$

$y = 0$

For x = 3:

$y = 5(3)^3((3)-4)$

$y = 5 \times 27 \times -1$

$y = -135$

SP's are (0,0) and (3,-135)

Example 1

Determine the coordinates and nature of the stationary points on the curve $y = 5x^3(x-4)$

Solution:

6. Draw a **NATURE TABLE** for each SP to determine its nature.

$$\frac{dy}{d(-1)} = 20(-1)^3 - 60(-1)^2 = -80$$

$$\frac{dy}{d(1)} = 20(1)^3 - 60(1)^2 = -40$$

7. Make statement

SP's are (0,0) and (3,-135)

(0,0) → $\frac{dy}{dx} = 20x^3 - 60x^2$

X	0^-	0	0^+
$\frac{dy}{dx}$	-ve	0	-ve
Slope	↘	—	↘

(0,0) is a falling point of inflexion.

Example 1

Determine the coordinates and nature of the stationary points on the curve $y = 5x^3(x-4)$

Solution:

6. Draw a **NATURE TABLE** for each SP to determine its nature.

$$\frac{dy}{d(1)} = 20(1)^3 - 60(1)^2 = -40$$

$$\frac{dy}{d(4)} = 20(4)^3 - 60(4)^2 = 320$$

7. Make statement

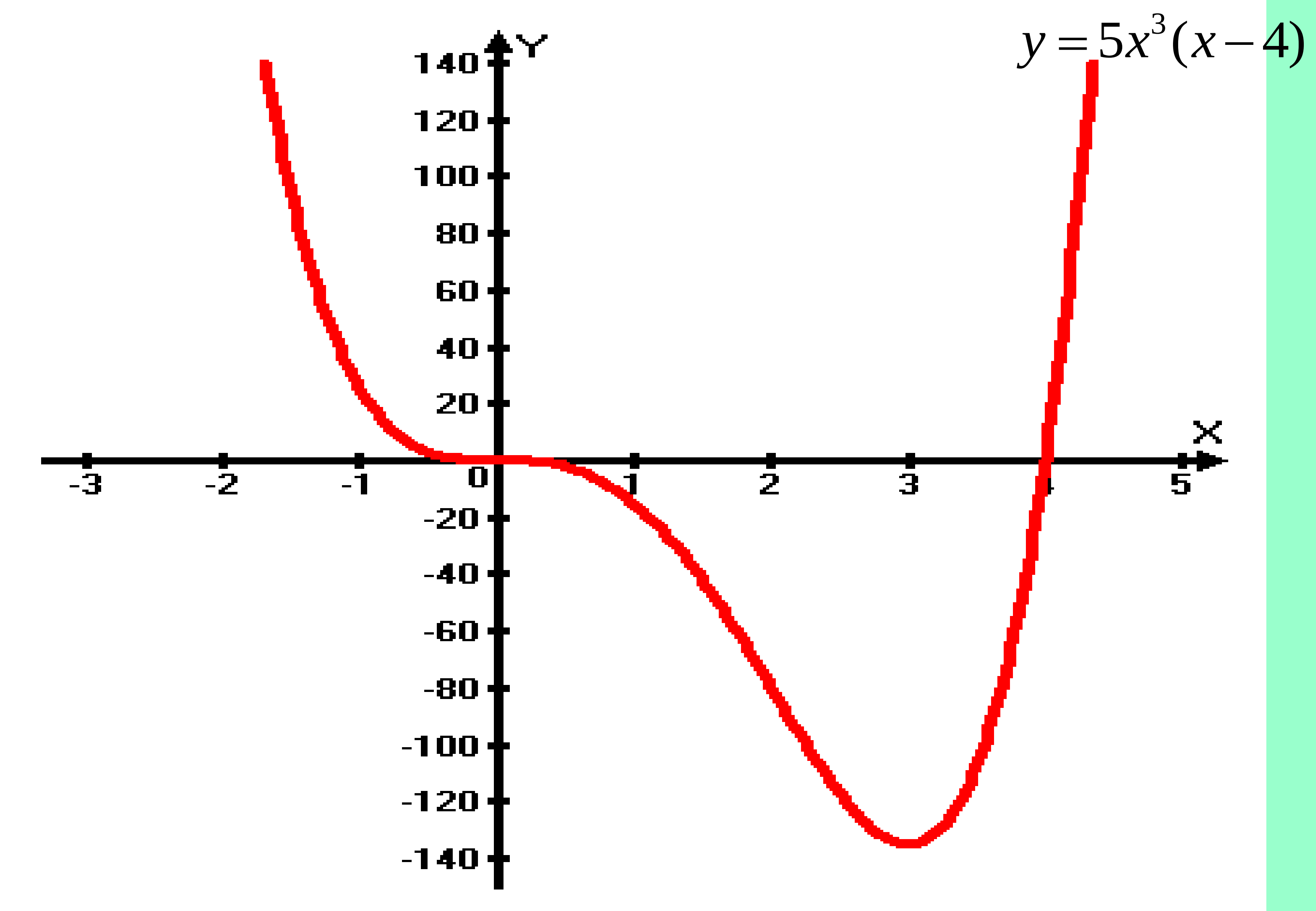
SP's are (0,0) and (3,-135)

(3,-135) ⟶ $\frac{dy}{dx} = 20x^3 - 60x^2$

X	3^-	3	3^+
$\frac{dy}{dx}$	-ve	0	+ve
Slope	\	—	/

(3,-135) is a minimum turning point.

The Proof


$y = 5x^3(x-4)$
Y
X
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-3
-2
-1
1
2
3
5

Heinemann , p.106, EX 6M