

7. More Complex Log Graphs



Extending Unit 1

In unit 1 we looked at transformations of graphs and it was pointed out that all the transformations which applied to normal curves also applied to trig, log and exponential curves.

Now that we have learned the rules for logs we will look at how to apply these rules to draw slightly more complex curves than you met in Unit 1.

The essential thing to remember when graphing log functions is the same as with differentiation; **you must prepare the function. So there should be no fractions and no multiples of x i.e no $2x$, $3x$, $4x$ etc.**

You use the laws of logs you have now learnt to get rid of these problems.

Example 1

(a) Sketch the graph of $f(x) = \log_3 \left(\frac{1}{x} \right)$

Powers to front & multiply

(b) Now sketch the graph of $f(x) = \log_3 (3x)$

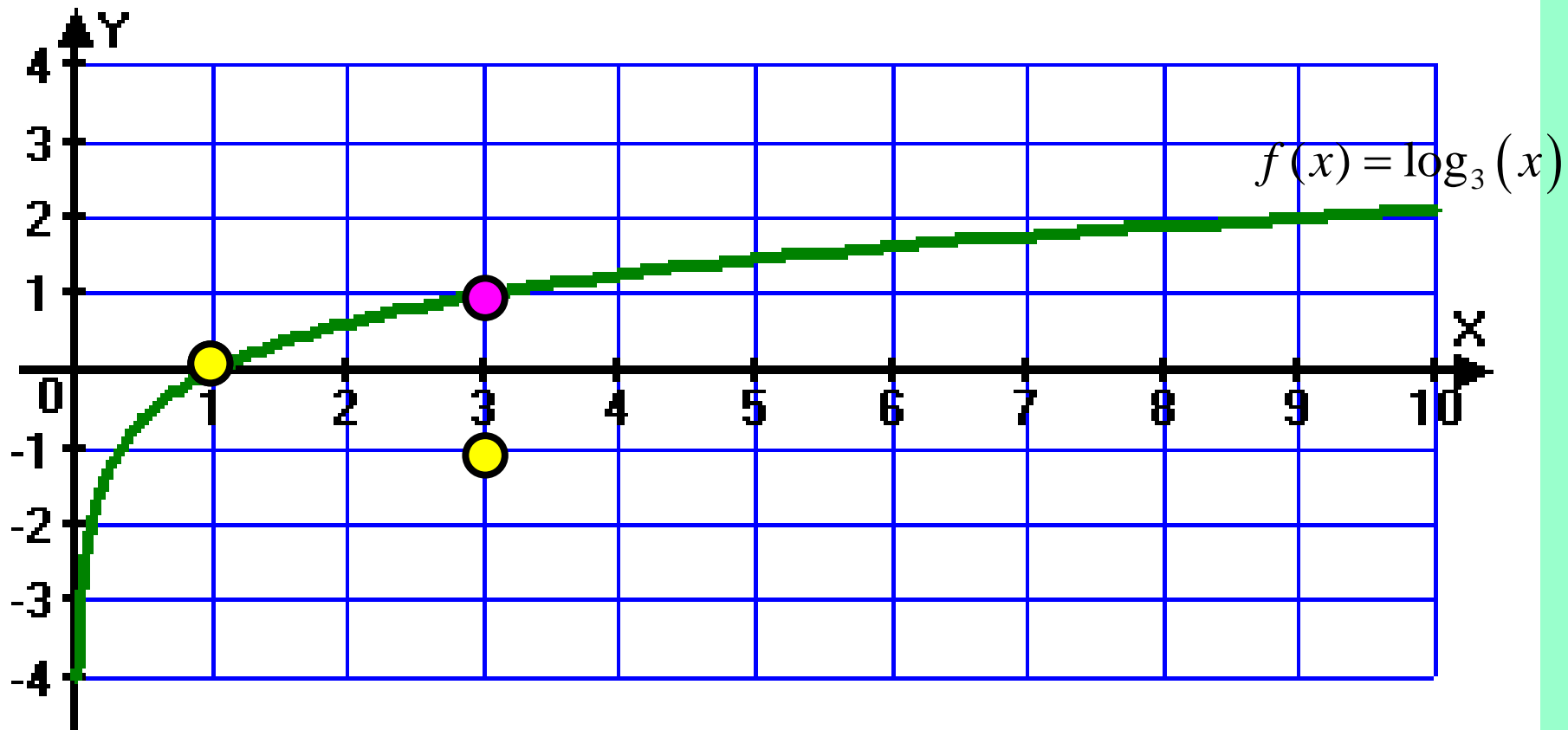
Solution to (a):

Must prepare: get rid of fractions and multiples of x by using the laws of logs.

$$f(x) = \log_3 \left(\frac{1}{x} \right) \longrightarrow f(x) = \log_3 (x^{-1})$$

$$f(x) = -\log_3 (x)$$

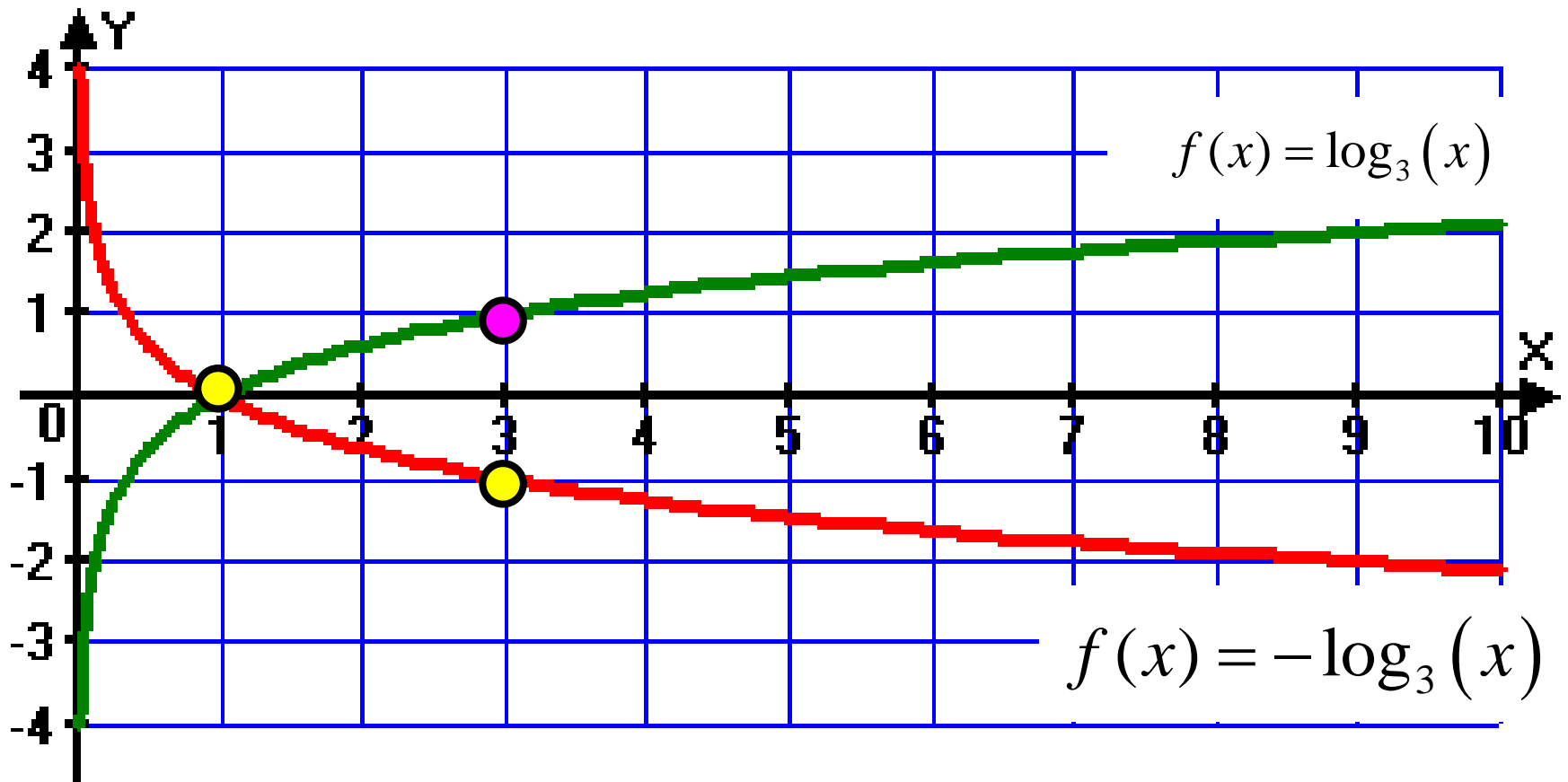
This is $\log_3 x$ reflected in x-axis.



$f(x) = -\log_3(x)$ → This is $\log_3 x$ reflected in x-axis.

Remember graph of $\log_3 x$ must pass through $(1, 0)$ and $(3, 1)$

So $f(x) = -\log_3(x)$ must pass through $(1, 0)$ and $(3, -1)$



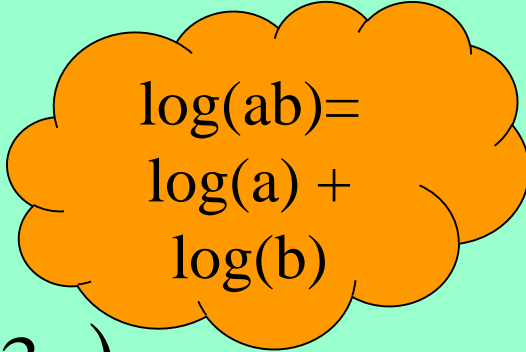
$f(x) = -\log_3(x)$ → This is $\log_3 x$ reflected in x-axis.

Remember graph of $\log_3 x$ must pass through (1,0) and (3,1)

So $f(x) = -\log_3(x)$ must pass through (1, 0) and (3, -1)

Example 1

(a) Sketch the graph of $f(x) = \log_3 \left(\frac{1}{x} \right)$


$$\log(ab) = \log(a) + \log(b)$$

(b) Now sketch the graph of $f(x) = \log_3(3x)$

Solution to (b):

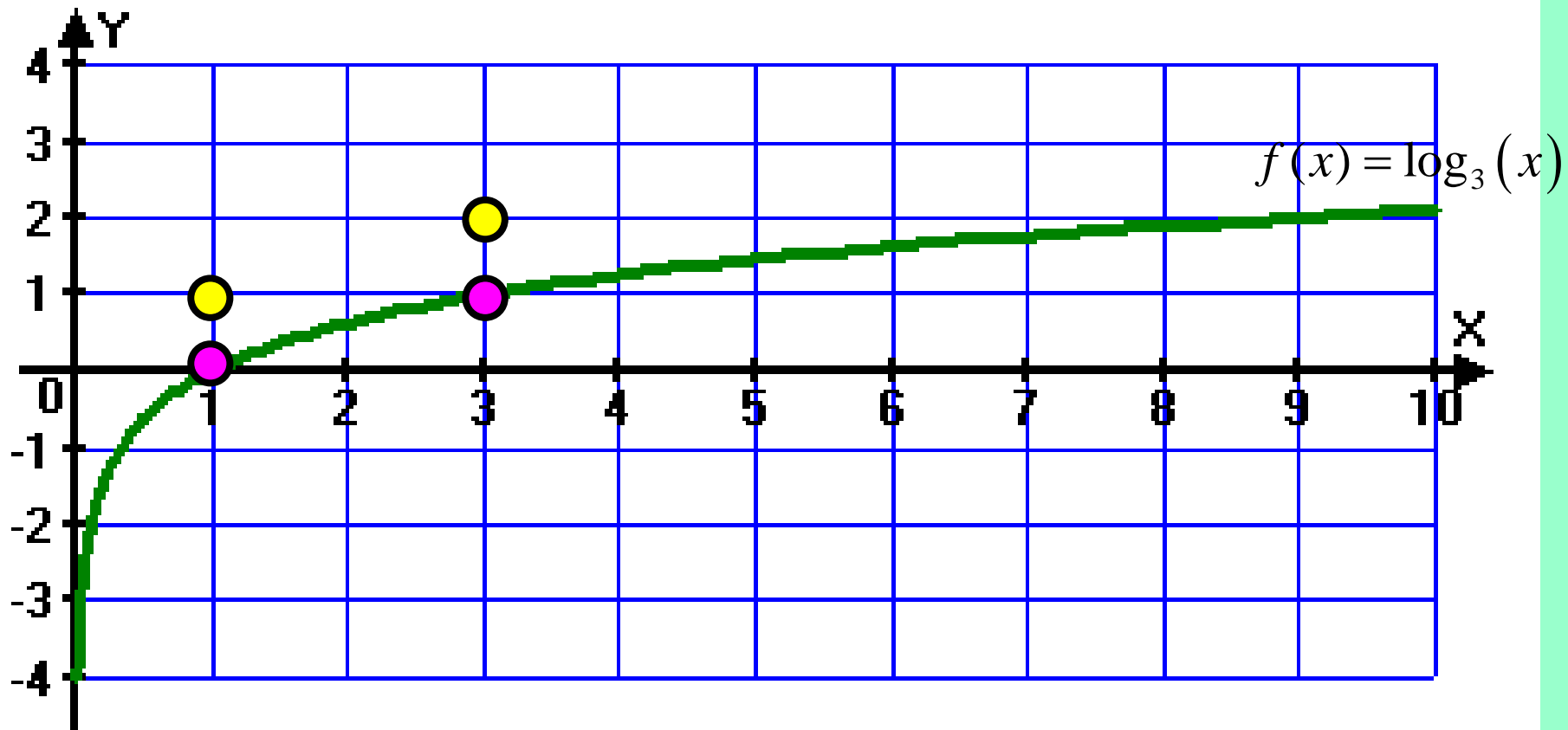
Must prepare: get rid of fractions and multiples of x by using the laws of logs.

$$f(x) = \log_3(3x) \longrightarrow f(x) = \log_3(3) + \log_3(x)$$

$$f(x) = 1 + \log_3(x)$$



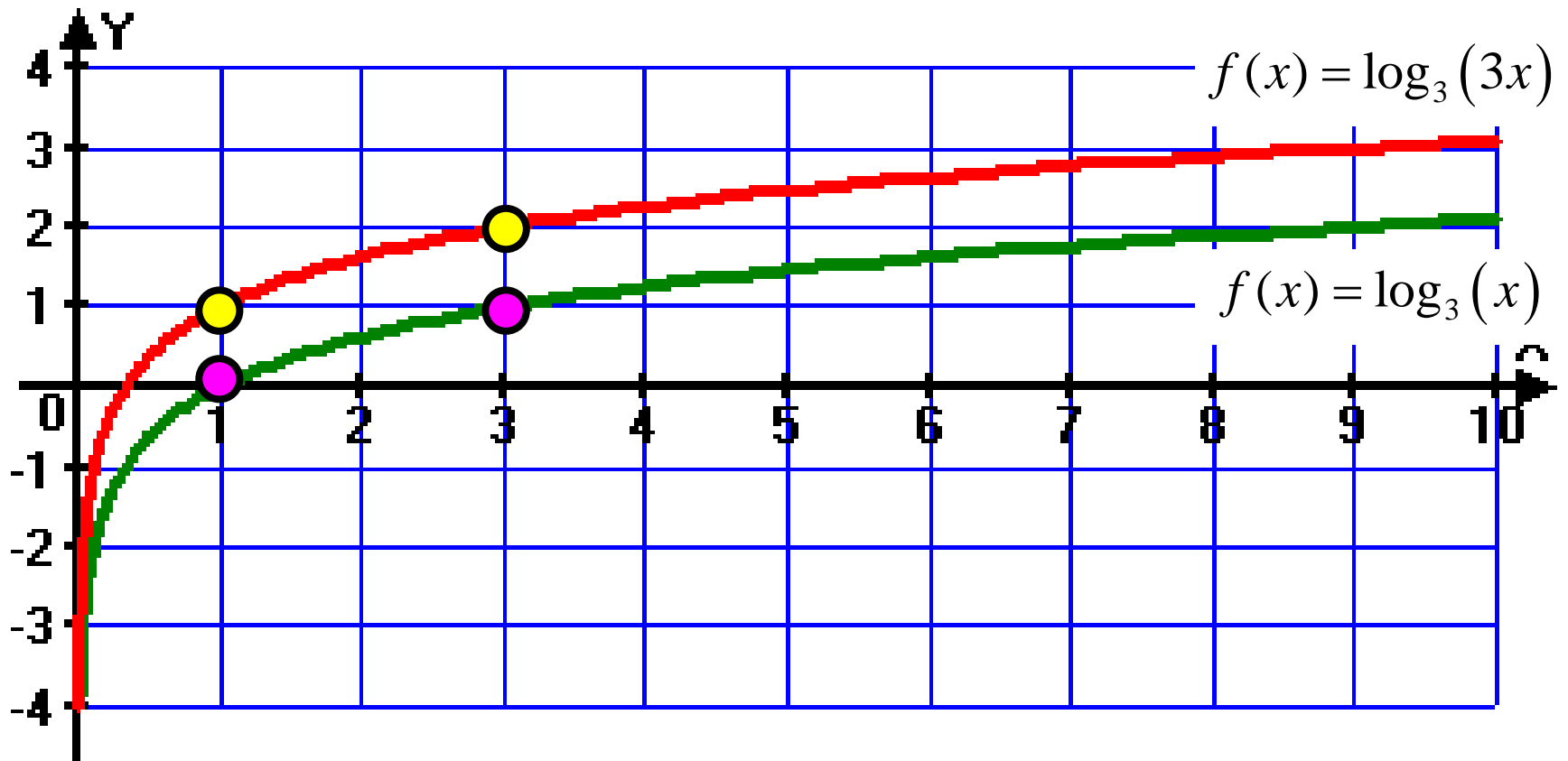
This is $\log_3 x$ moved up y-axis by 1



$f(x) = \log_3(3x)$ → This is $\log_3 x$ moved up y-axis by 1.

Remember graph of $\log_3 x$ must pass through (1,0) and (3,1)

So $f(x) = \log_3(3x)$ must pass through (1, 1) and (3, 2)



$f(x) = \log_3(3x)$ \longrightarrow This is $\log_3 x$ moved up y-axis by 1.

Remember graph of $\log_3 x$ must pass through (1,0) and (3,1)

So $f(x) = \log_3(3x)$ must pass through (1, 1) and (3, 2)

Heinemann, p.296, EX 15K,
Q1& 2

This is not the end


Example 2

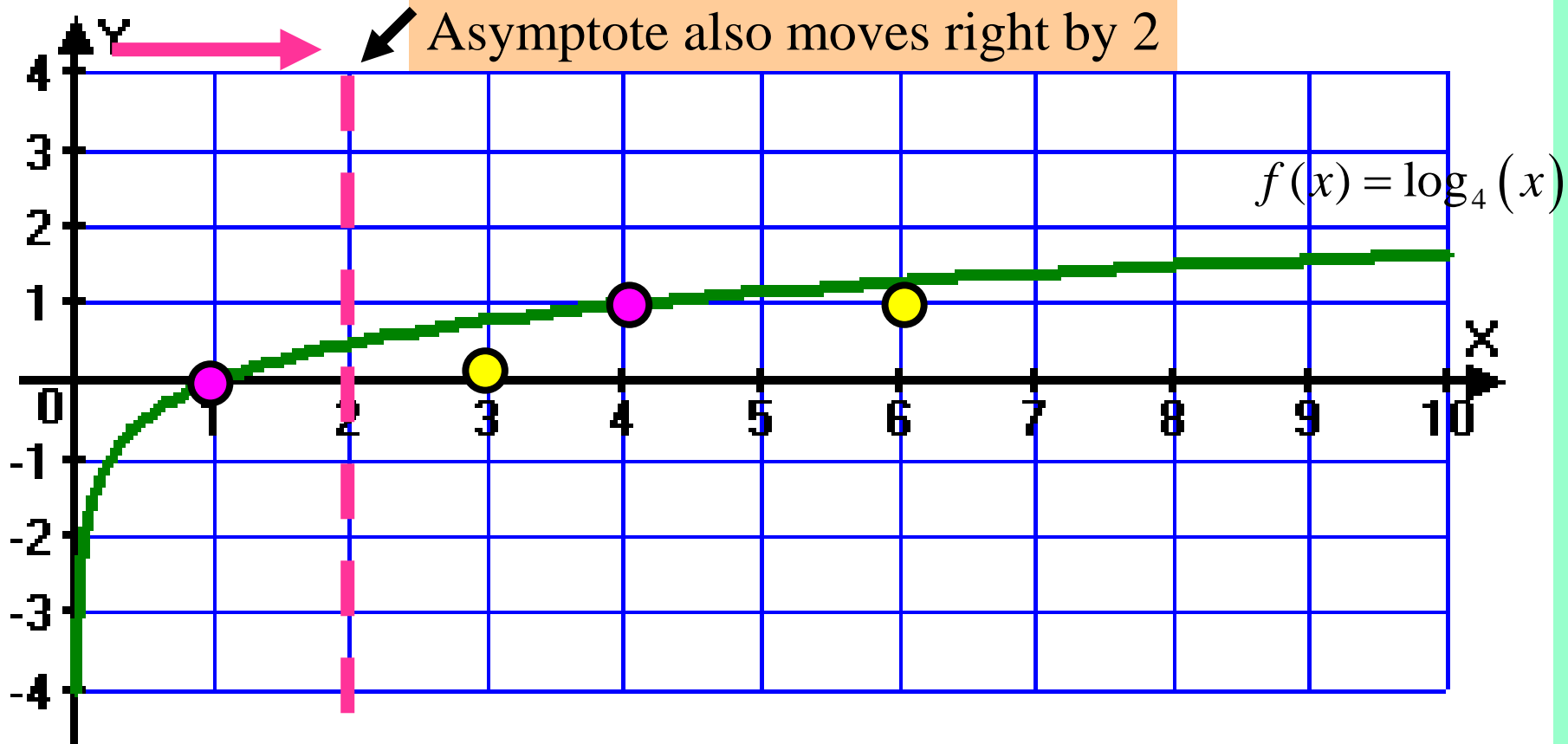
(a) Sketch the graph of $f(x) = \log_4(x - 2)$

(b) Now sketch the graph of $f(x) = \log_4 16(x - 2)$

Solution to (a):

Must prepare: get rid of fractions and multiples of x by using the laws of logs.

$f(x) = \log_4(x - 2)$  This is $\log_4 x$ moved right by 2.

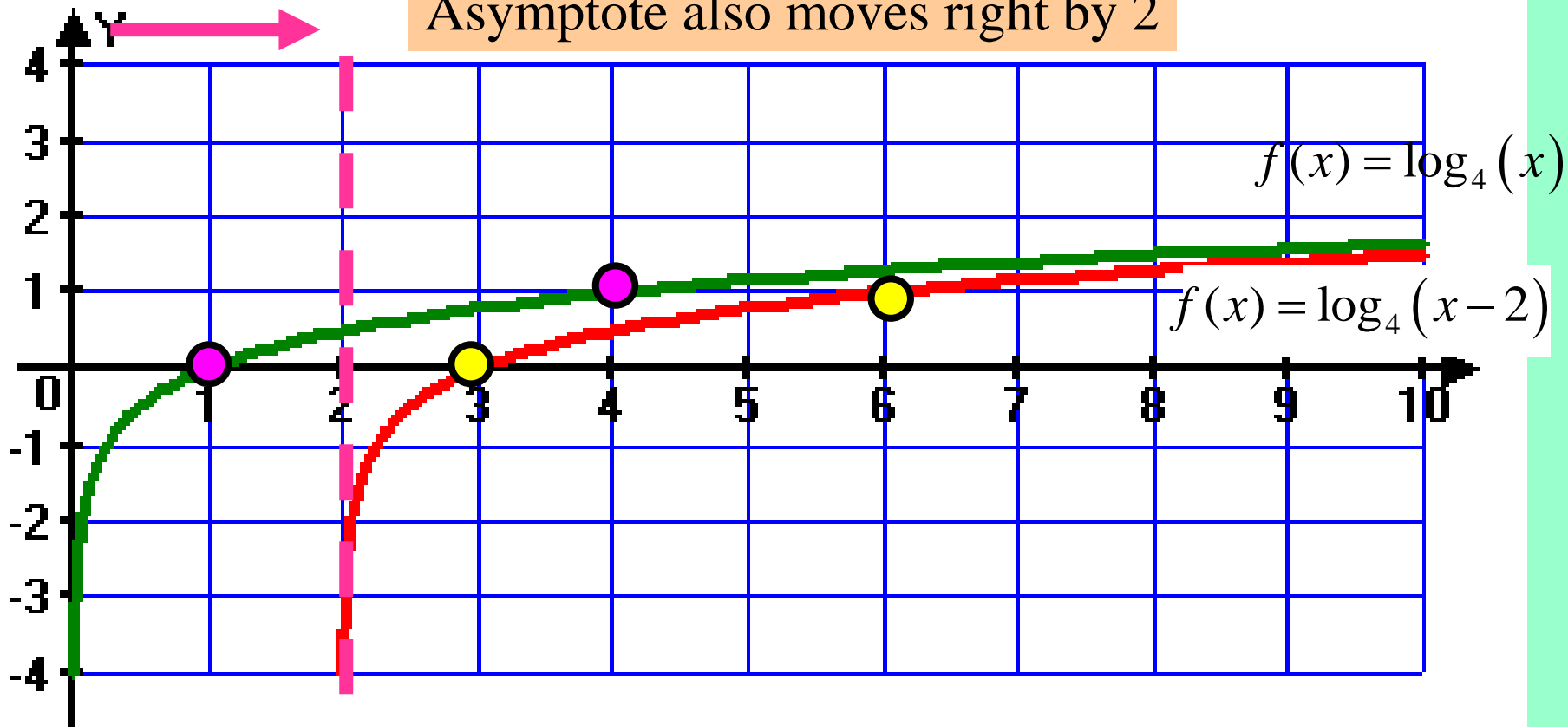


$$f(x) = \log_4(x - 2) \longrightarrow \text{This is } \log_4 x \text{ moved right by 2.}$$

Remember graph of $\log_4 x$ must pass through $(1, 0)$ and $(4, 1)$

So $f(x) = \log_4(x - 2)$ must pass through $(3, 0)$ and $(6, 1)$

Asymptote also moves right by 2



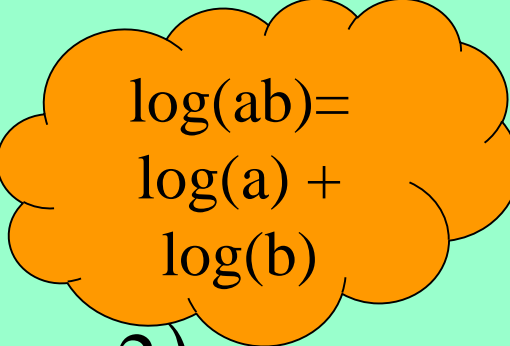
$f(x) = \log_4(x-2)$ → This is $\log_4 x$ moved right by 2.

Remember graph of $\log_4 x$ must pass through $(1, 0)$ and $(4, 1)$

So $f(x) = \log_4(x-2)$ must pass through $(3, 0)$ and $(6, 1)$

Example 2

(a) Sketch the graph of $f(x) = \log_4(x-2)$

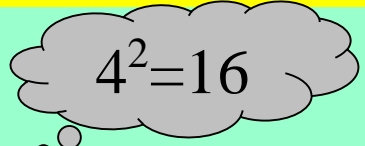

$$\log(ab) = \log(a) + \log(b)$$

(b) Now sketch the graph of $f(x) = \log_4 16(x-2)$

Solution to (b):

Must prepare: get rid of fractions and multiples of x by using the laws of logs.

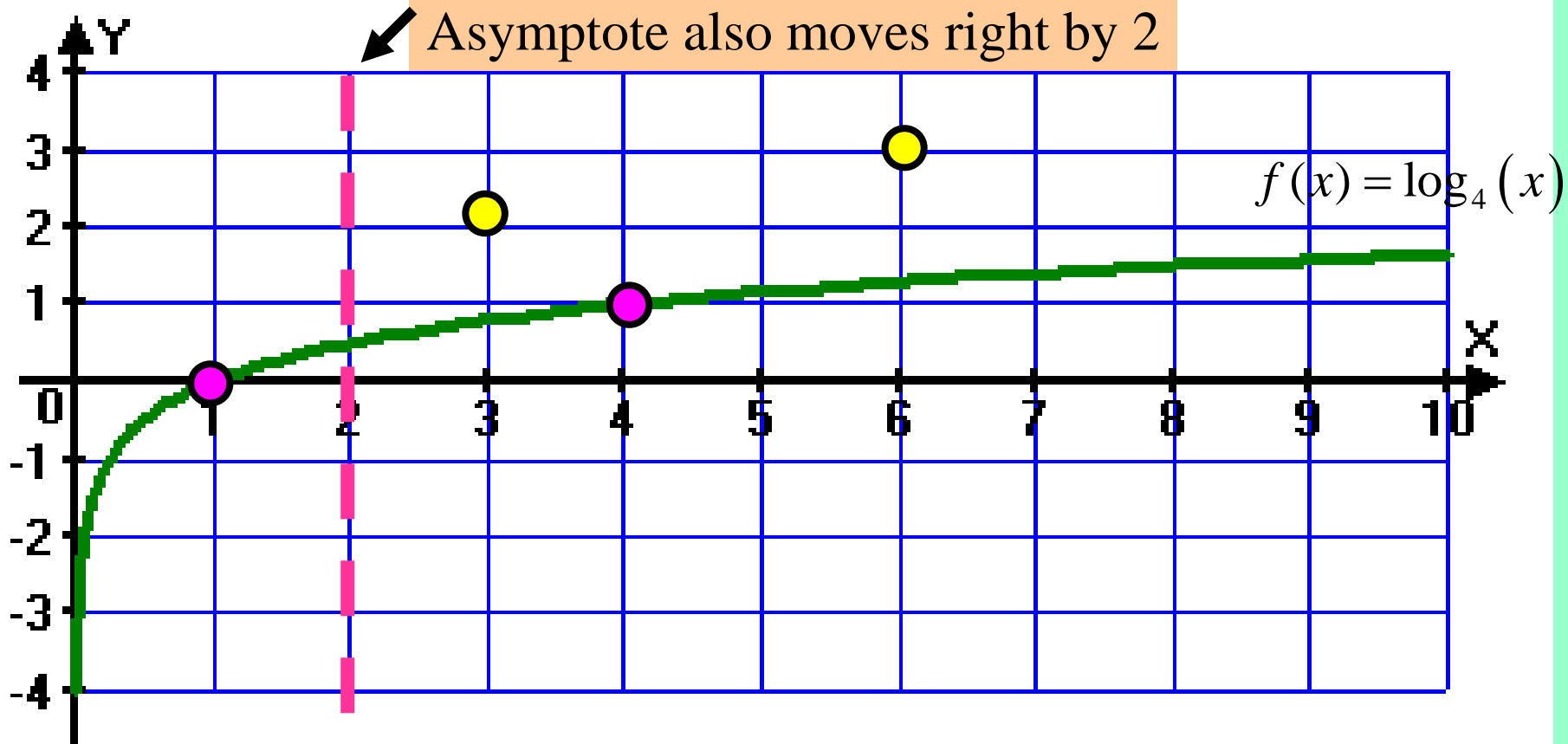
Would produce multiple of x!


$$4^2 = 16$$

$$f(x) = \log_4 16(x-2) \longrightarrow f(x) = \log_4(16) + \log_4(x-2)$$

$$f(x) = 2 + \log_4(x-2)$$

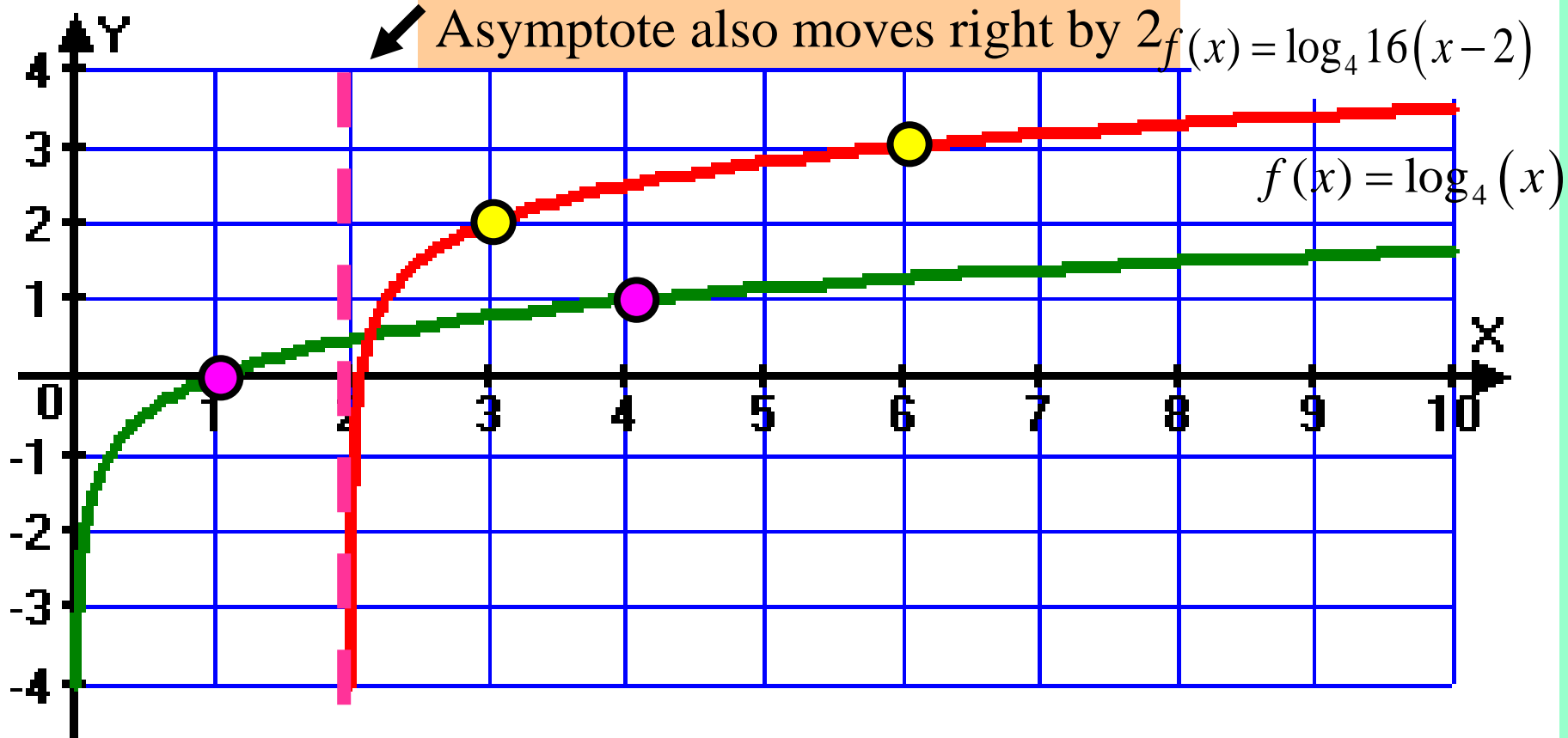
This is $\log_4 x$ moved right by 2 then up by 2.



$f(x) = \log_4 16(x - 2)$ → This is $\log_4 x$ moved right by 2 and up 2.

Remember graph of $\log_4 x$ must pass through (1,0) and (4,1)

So $f(x) = \log_4 16(x - 2)$ must pass through (3, 2) and (6, 3)



$f(x) = \log_4 16(x-2)$ → This is $\log_4 x$ moved right by 2 and up 2.

Remember graph of $\log_4 x$ must pass through (1,0) and (4,1)

So $f(x) = \log_4 16(x-2)$ must pass through (3, 2) and (6, 3)

Heinemann, p.296, EX 15K,
Q3 & 4