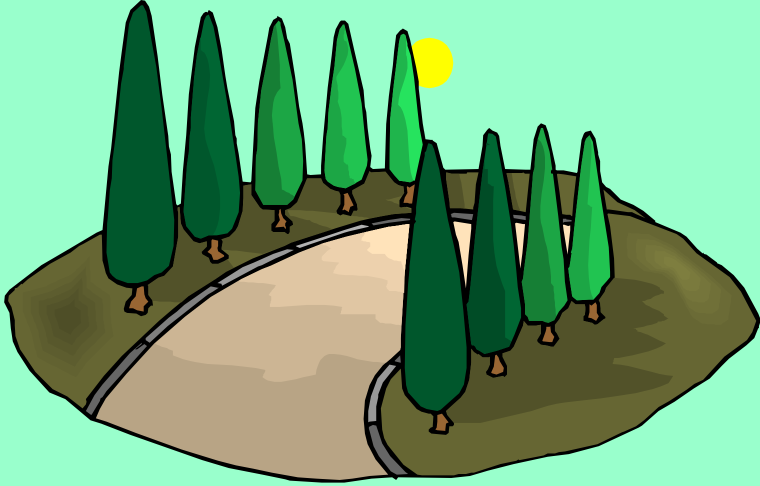
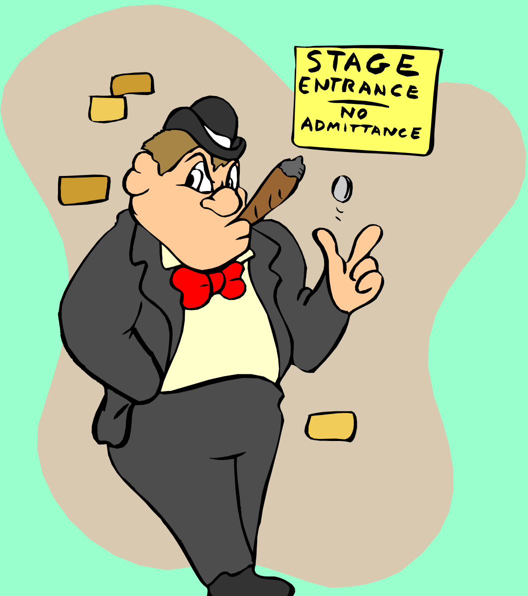




# 7. Logarithmic Functions

$$y = \log_a x$$



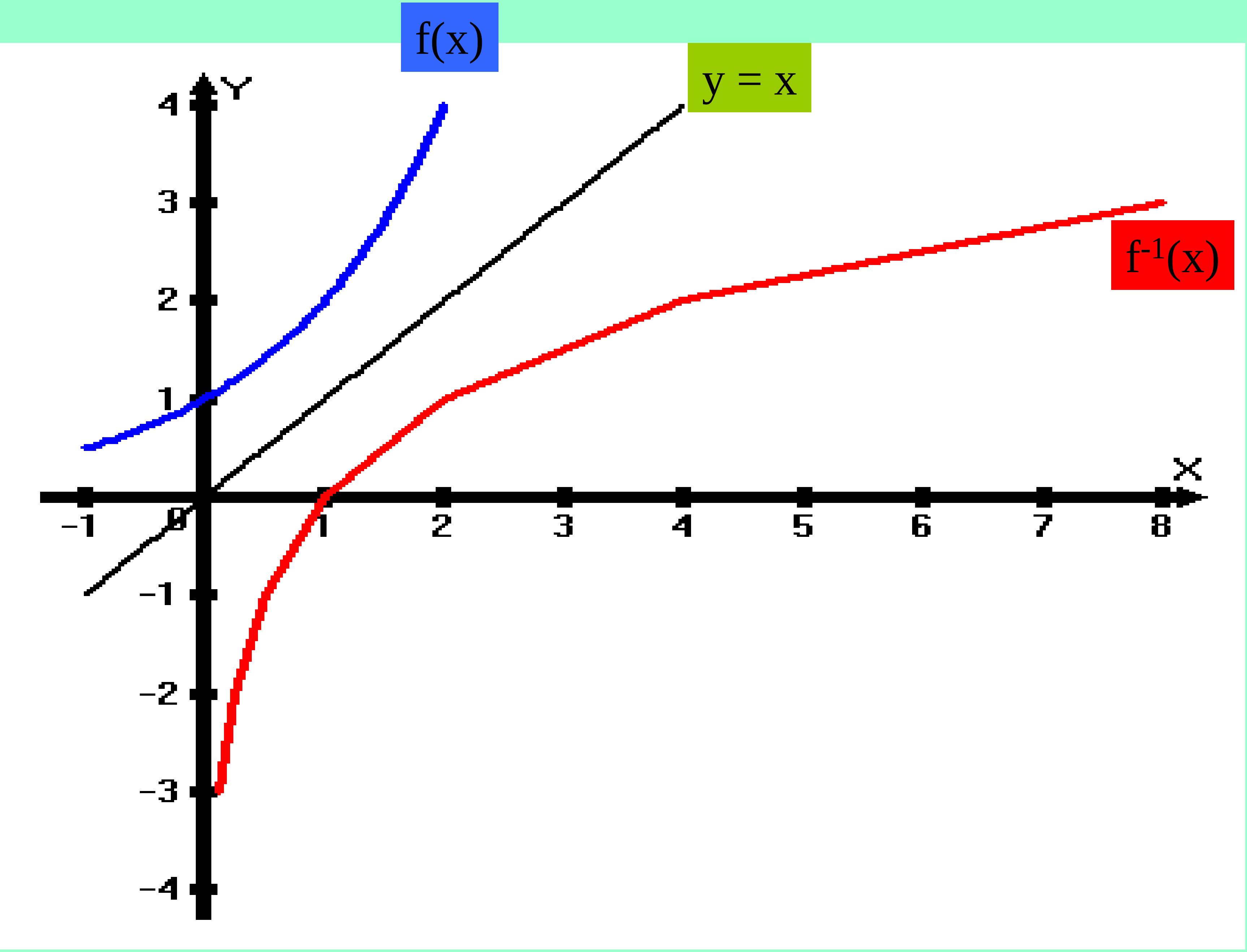
# The Logarithmic Function

Recall that to find the inverse of a function we reflect that function in the line y = x.

In essence this means **swapping the x and y co-ordinates**. For $f(x) = 2^x$ this leads to:

| **x** | **$^1/_8$** | **¼** | **½** | **1** | **2** | **4** | **8** |
|---|---|---|---|---|---|---|---|
| **$f^{-1}(x)$** | **-3** | **-2** | **-1** | **0** | **1** | **2** | **3** |

Plotting these points and $f(x) = 2^x$ on the same graph along with the line y = x we can see they are inverses:



f(x)
y = x
f-1(x)
Y
X
4
3
2
1
0
−1
−2
−3
−4
−1
1
2
3
4
5
6
7
8

## The Logarithmic Function

Copy the following :

The inverse of the exponential function is the logarithmic function.

This has the form:

"log base a of x"

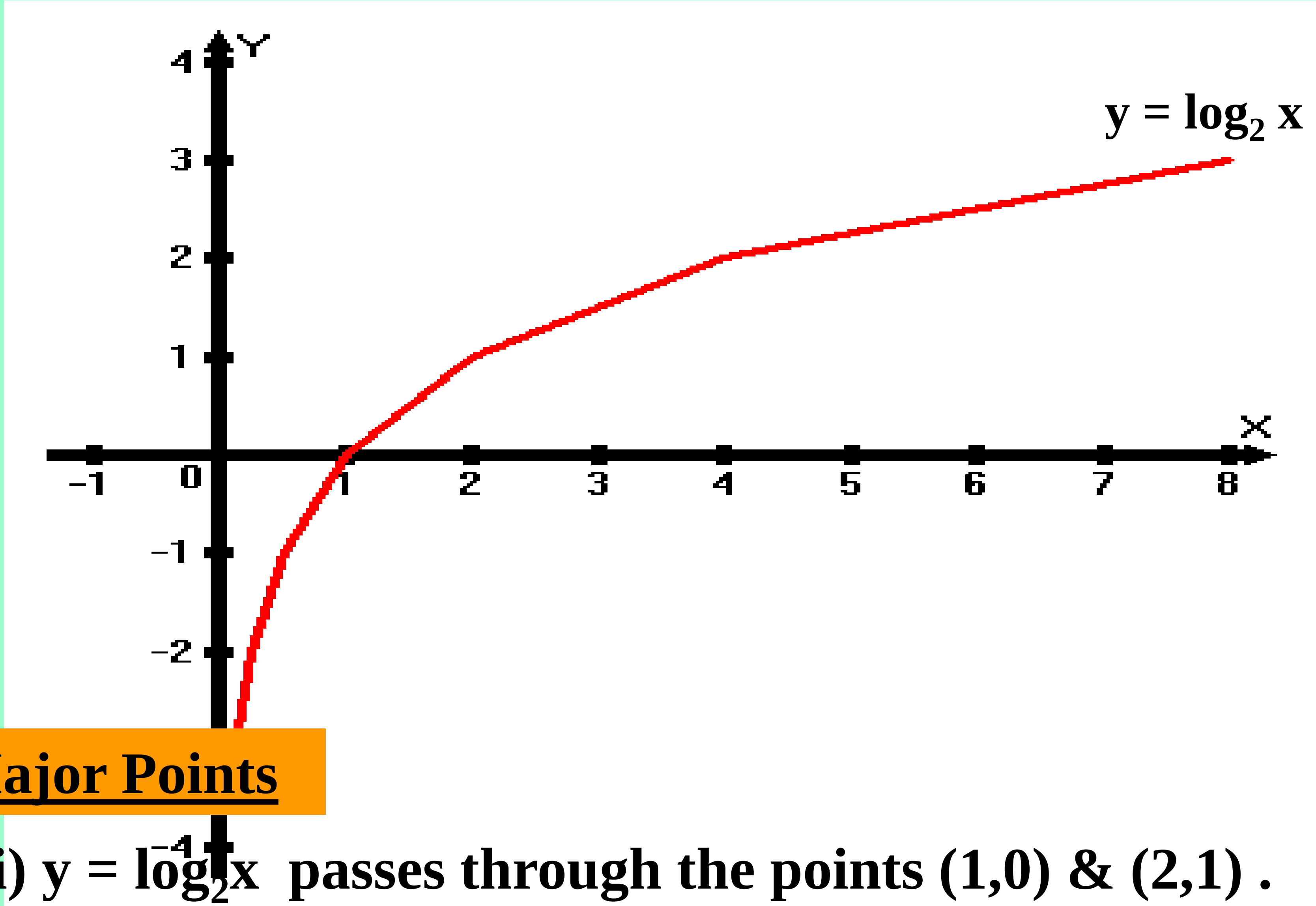
$$f(x) = \log_a x$$

X must be positive

When dealing with log functions we must think:

**"the base to what power gives x"** **or** $a^y = x$

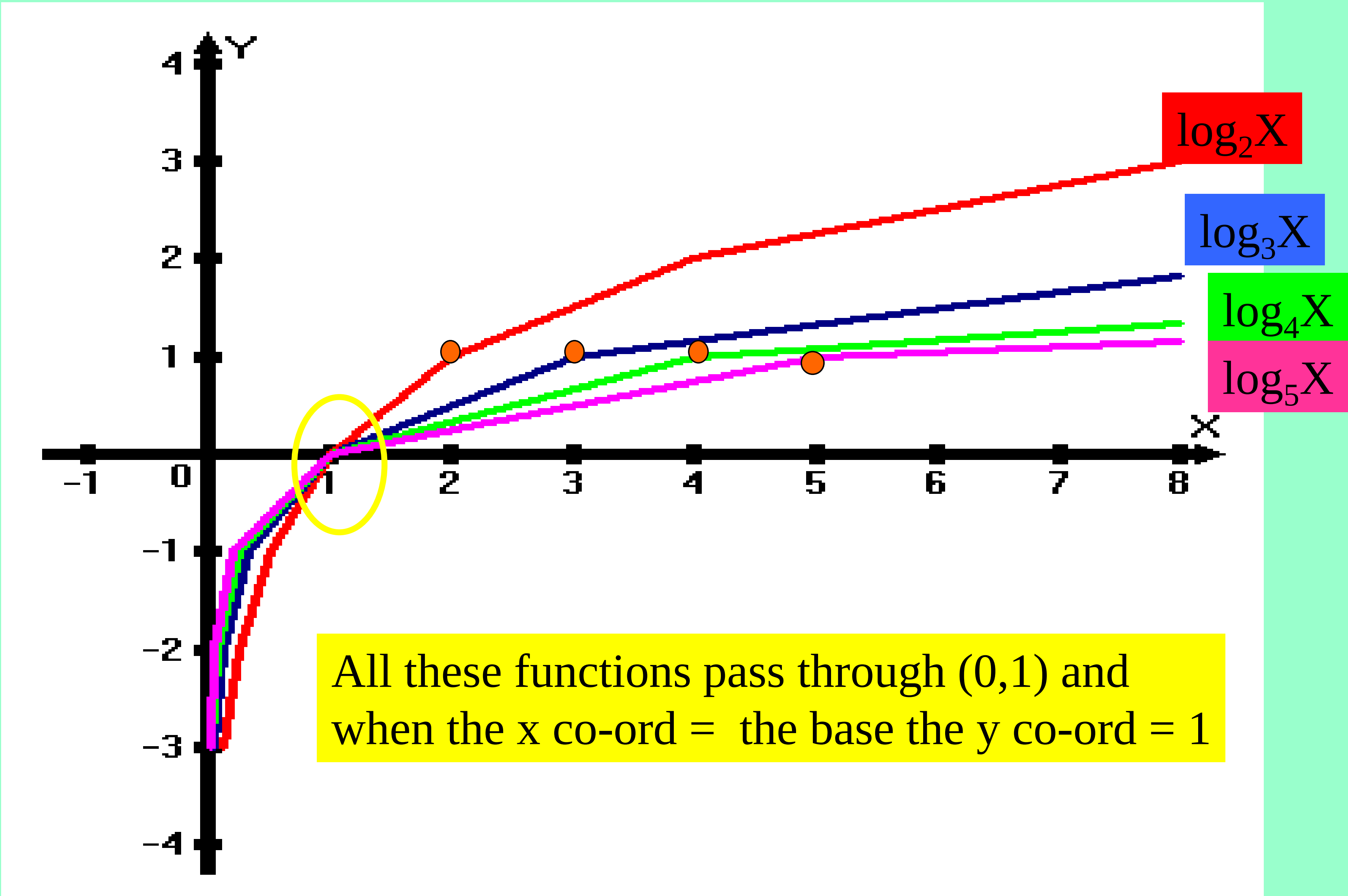
Lets look again at the graph of the inverse of f(x) = $2^x$




**Major Points**

(i) $y = \log_2 x$ passes through the points (1,0) & (2,1) .

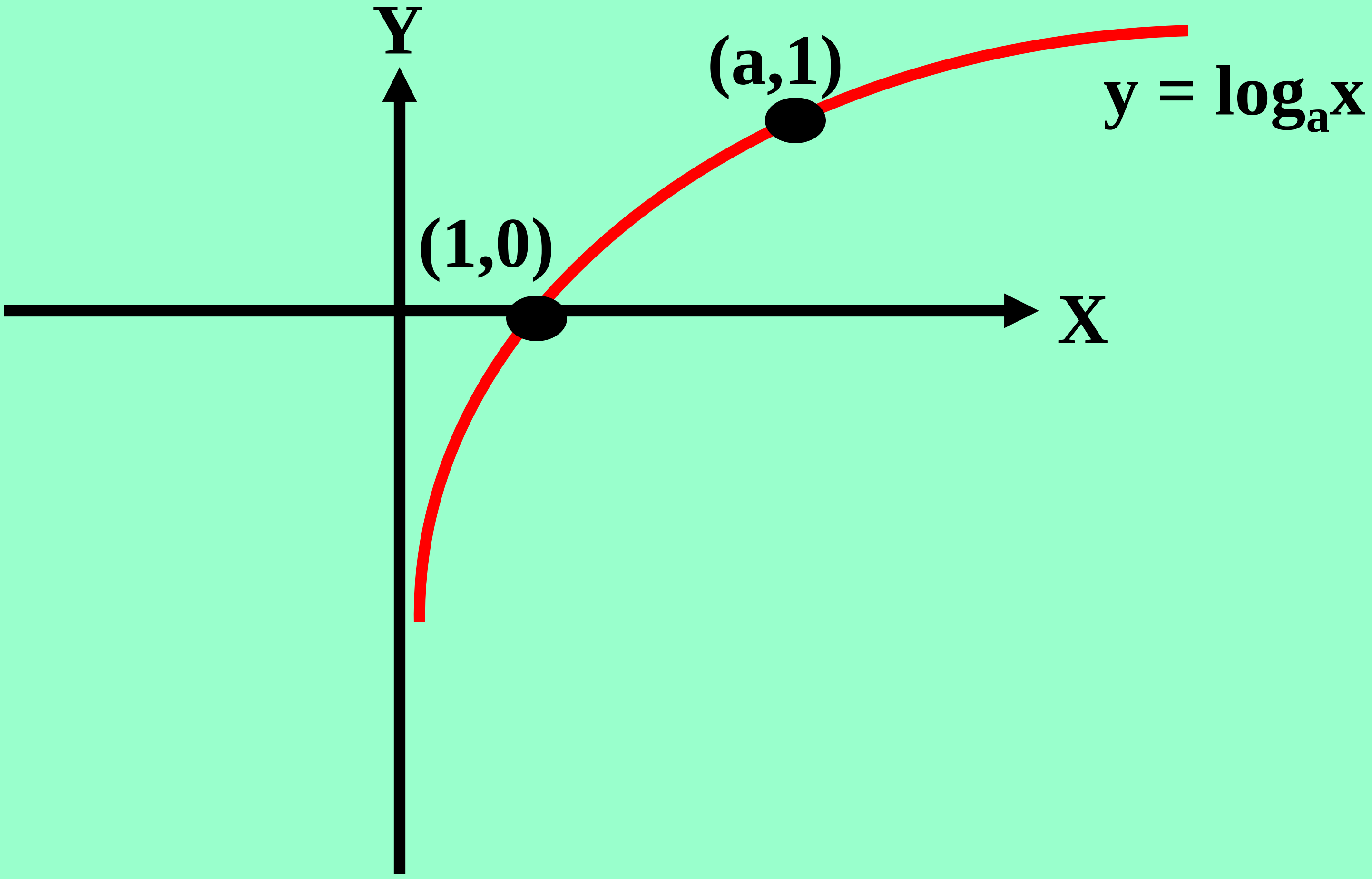
(ii) As $x \to \infty$ $y \to \infty$ but at a very slow rate and as $x \to 0$ $y \to -\infty$ .


Y
X
4
3
2
1
0
−1
−2
−3
−4
−1
1
2
3
4
5
6
7
8
$\log_2 X$
$\log_3 X$
$\log_4 X$
$\log_5 X$
All these functions pass through (0,1) and
when the x co-ord = the base the y co-ord = 1

Copy the following :

**The graph of $y = \log_a x$ always passes through (1,0) & (a,1)**

**It looks like ..**



Example 1

Write down the inverse of the function $y = 3^x$

Solution:

Inverse of exponential function y = $a^x$ is y = $\log_a x$

So inverse is : $y = \log_3 x$

# Example 2

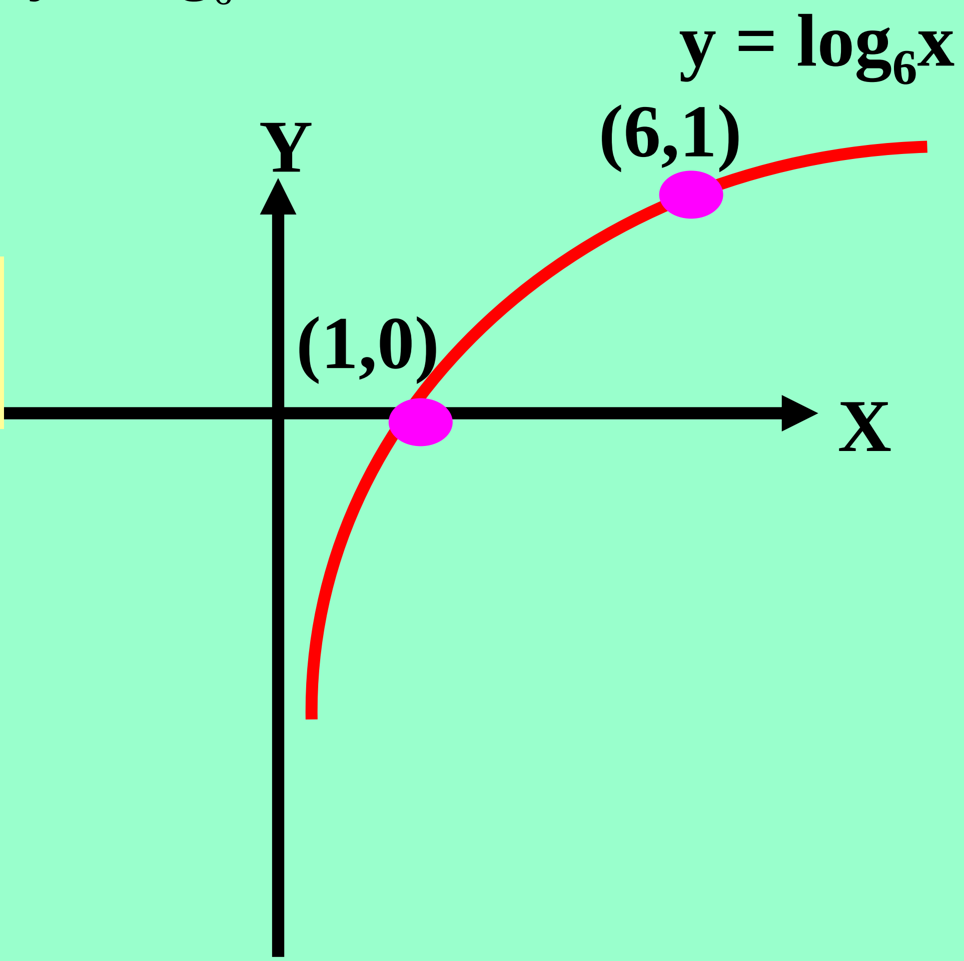
Sketch the graph of the function $y = \log_6 x$

**Solution:**

1. This graph must pass through (1 , 0) and (6 , 1)

2. Plot these points

3. Join the points with a smooth curve.

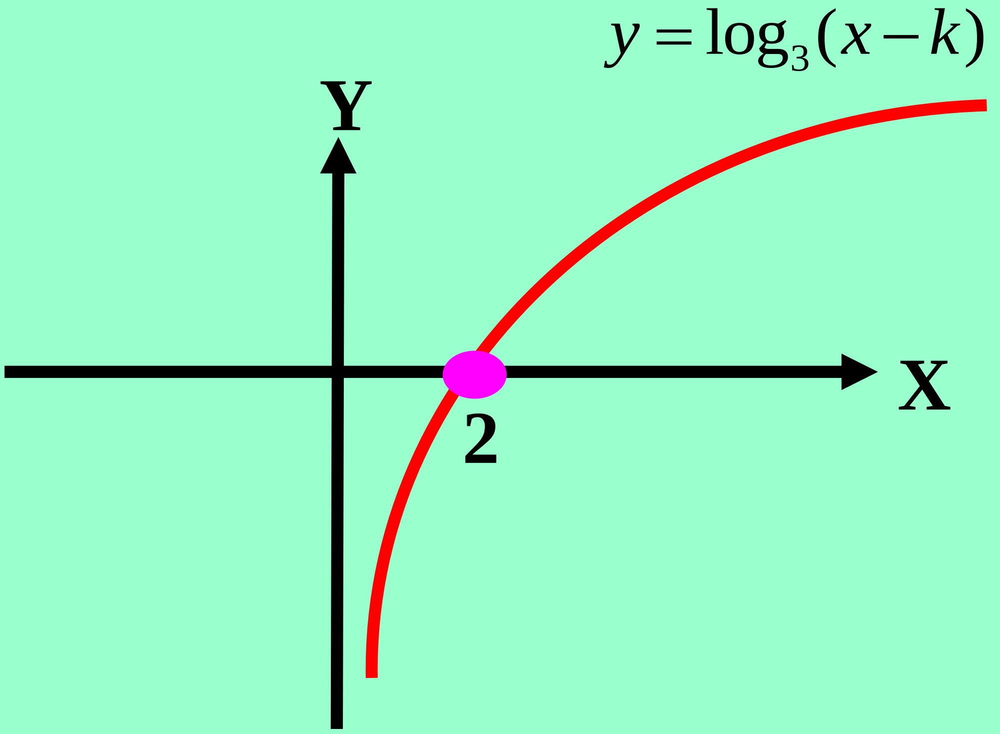


Example 3

This graph is in the form

$$y = \log_3(x - k)$$

Find the value of k.




Solution:

1. Write down form $a^y = x$

$$3^y = (x - k)$$

2. Substitute in coords of given point

For (2,0) → $3^0 = (2 - k)$

$1 = 2 - k$

$k = 1$

Heinemann, p.47, EX 3O,
NOT Q2(b) & (c)