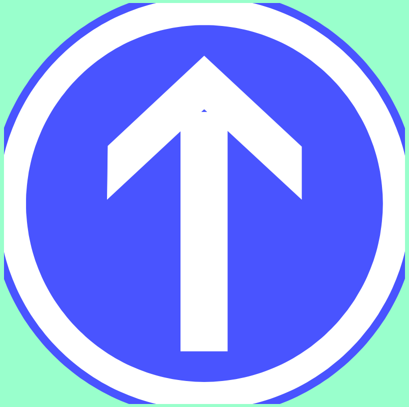


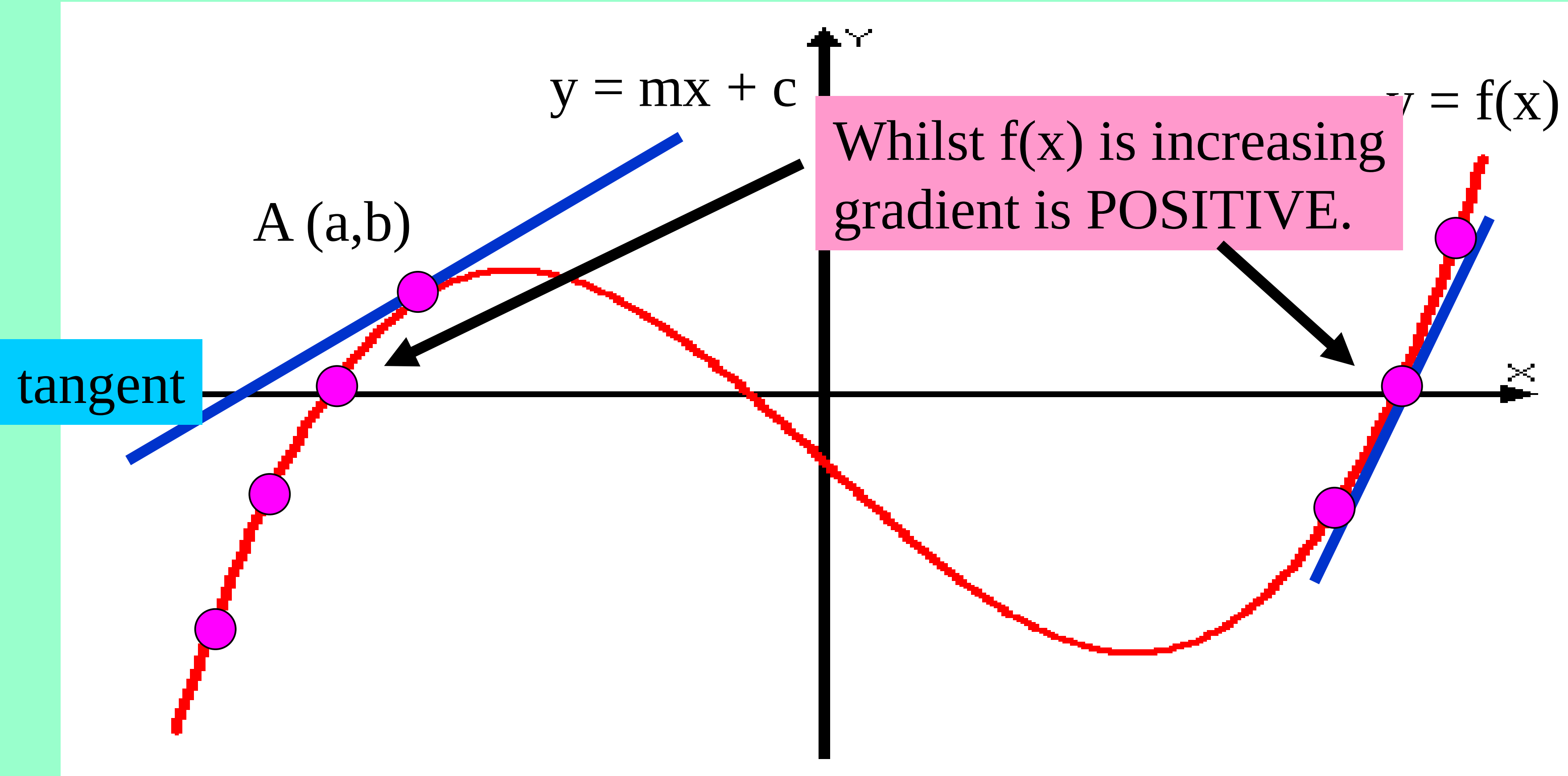
7.

Increasing / Decreasing Functions

$$f'(x) > 0$$

$$f'(x) < 0$$

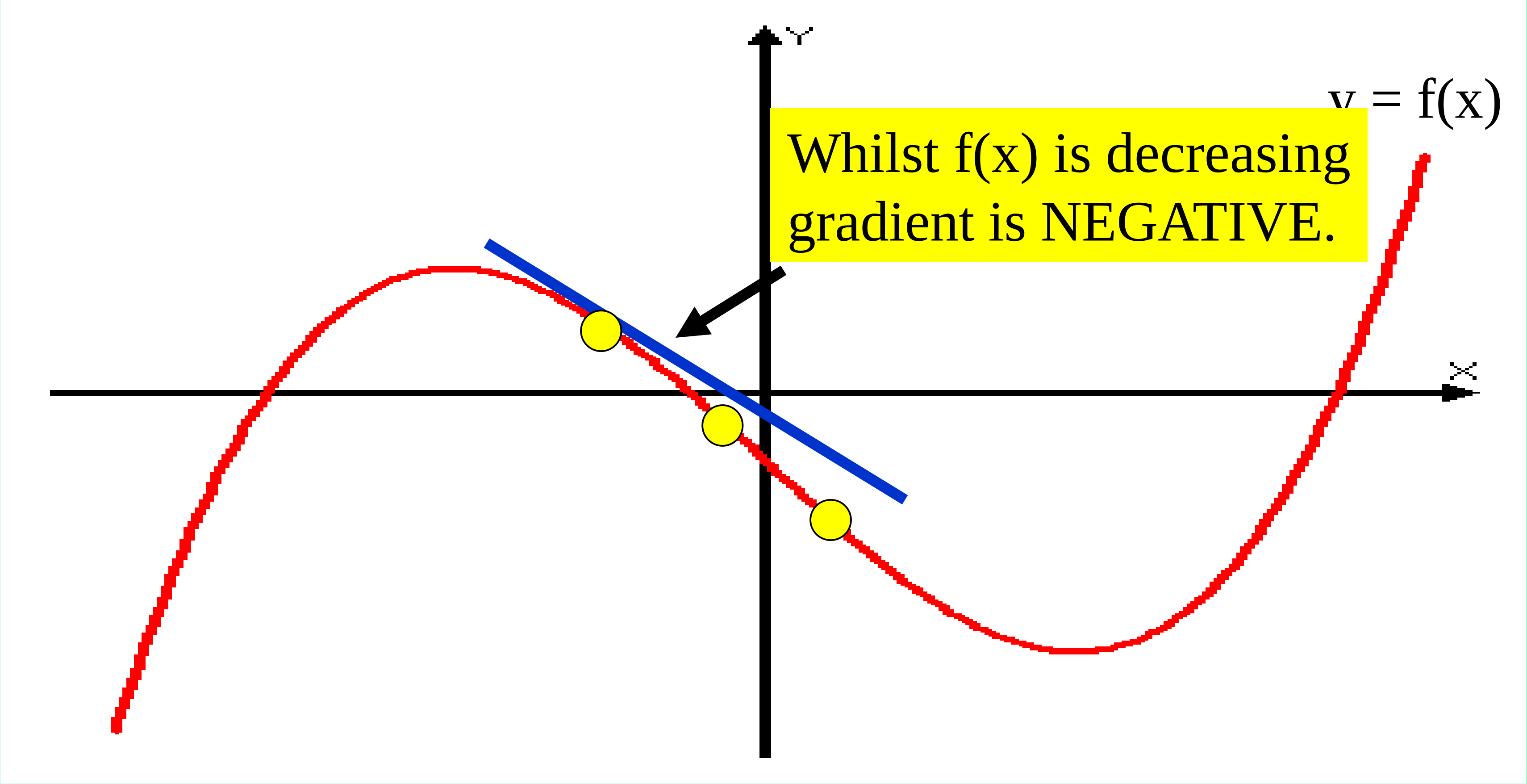
<u>Is the function increasing or decreasing?</u>



Our next task is to establish whether or not a function is increasing or decreasing.

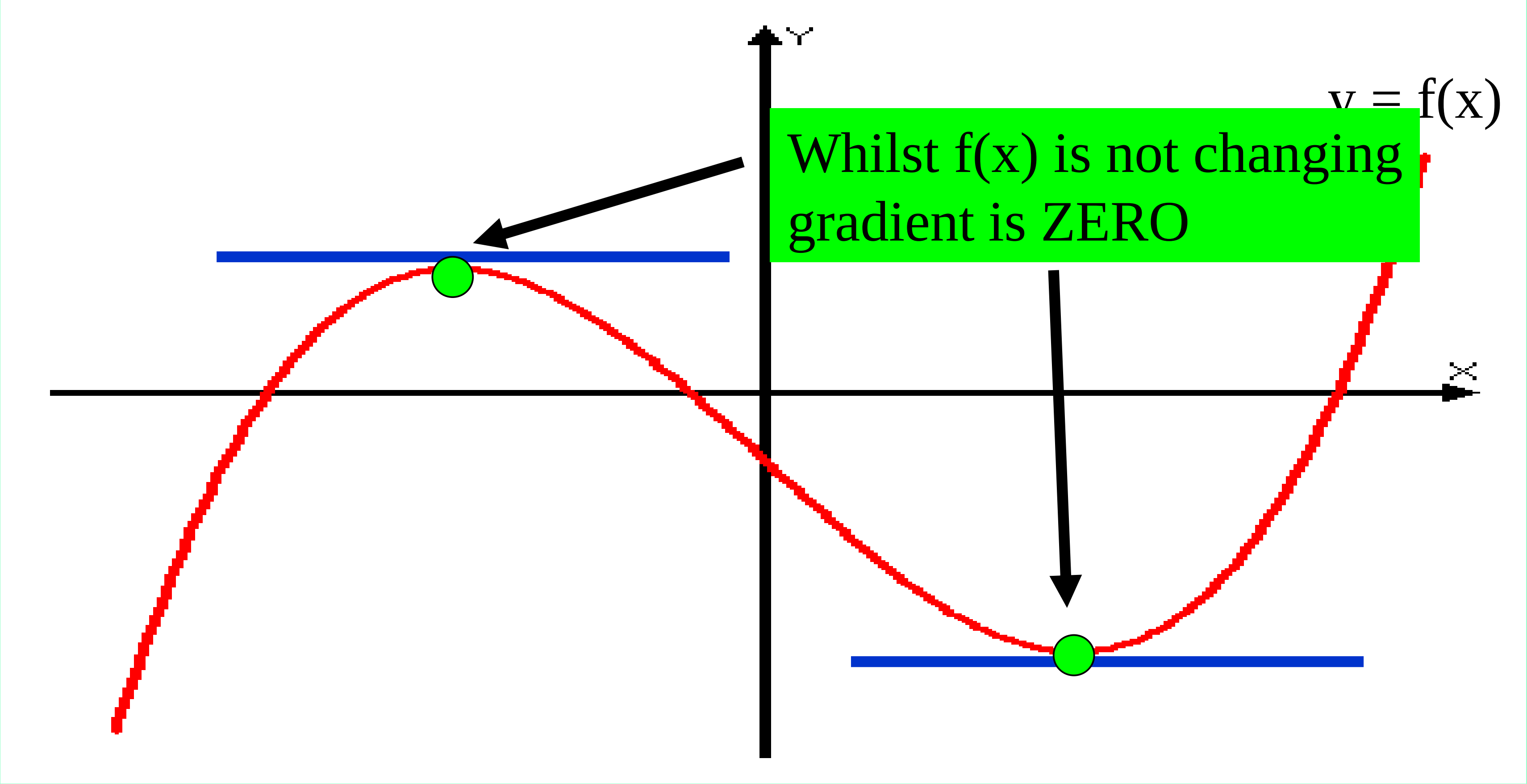
What can we say about the gradient of the tangent as the y- coordinates **increase** i.e. the value of the function is increasing?

<u>Is the function increasing or decreasing?</u>



What can we say about the gradient of the tangent as the y- coordinates **decrease** i.e. the value of the function is decreasing?

Is the function increasing or decreasing?



What happens when the graph is changing direction?

The tangent is horizontal and so the gradient is ZERO.

These are called **stationary points.**

Increasing and Decreasing Functions

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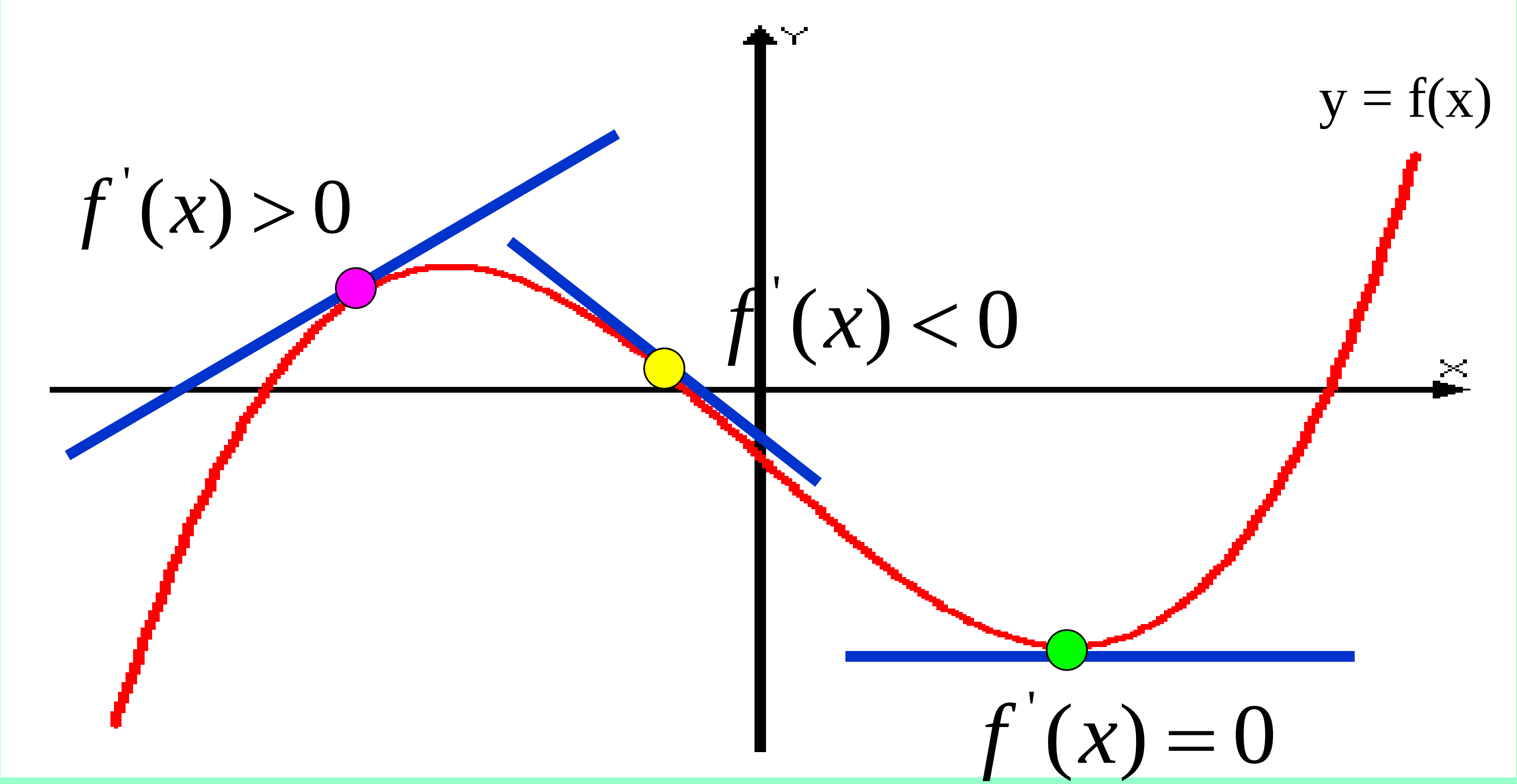


$f'(x) > 0$ → Function increasing

$f'(x) < 0$ → Function decreasing

$f'(x) = 0$ → Stationary Point

Example 1

State whether the function $y = 4x^3 + 3x^2 - 3$ is increasing or decreasing at x = -2

Solution:

1. Find dy / dx

$$y = 4x^3 + 3x^2 - 3$$

$$\frac{dy}{dx} = 12x^2 + 6x$$

2. Find gradient at x-coordinate

$$\frac{dy}{dx(-2)} = 12 \times (-2)^2 + 6 \times (-2)$$

$$\frac{dy}{dx(-2)} = 48 - 12 \quad = 36$$

3. Make statement

As dy/dx is positive, function is increasing at x = -2

Example 2

Find the intervals in which the function $y = 4x^3 + 6x^2 - 2$ is increasing and decreasing.

Solution:

1. Find dy/dx

$$y = 4x^3 + 6x^2 - 2$$

$$\frac{dy}{dx} = 12x^2 + 12x$$

2. Set dy/dx = 0 and solve to find x-coords of Stationary Points

$$12x^2 + 12x = 0$$

$$12x(x+1) = 0$$ Factorise

$$12x = 0 \quad \text{or} \quad x + 1 = 0$$

$$x = 0 \quad \text{or} \quad x = -1$$

3. Make a table to show what gradient is before and after SP's

X	→	-1	→	0	→
$\frac{dy}{dx}$	+	0	-	0	+

Example 2

Find the interval in which the function $y = 4x^3 + 6x^2 - 2$ is increasing and decreasing.

Solution:

4. Make statement

f(x) increasing: $x < -1$

f(x) decreasing: $-1 < x < 0$

f(x) increasing: $x > 0$

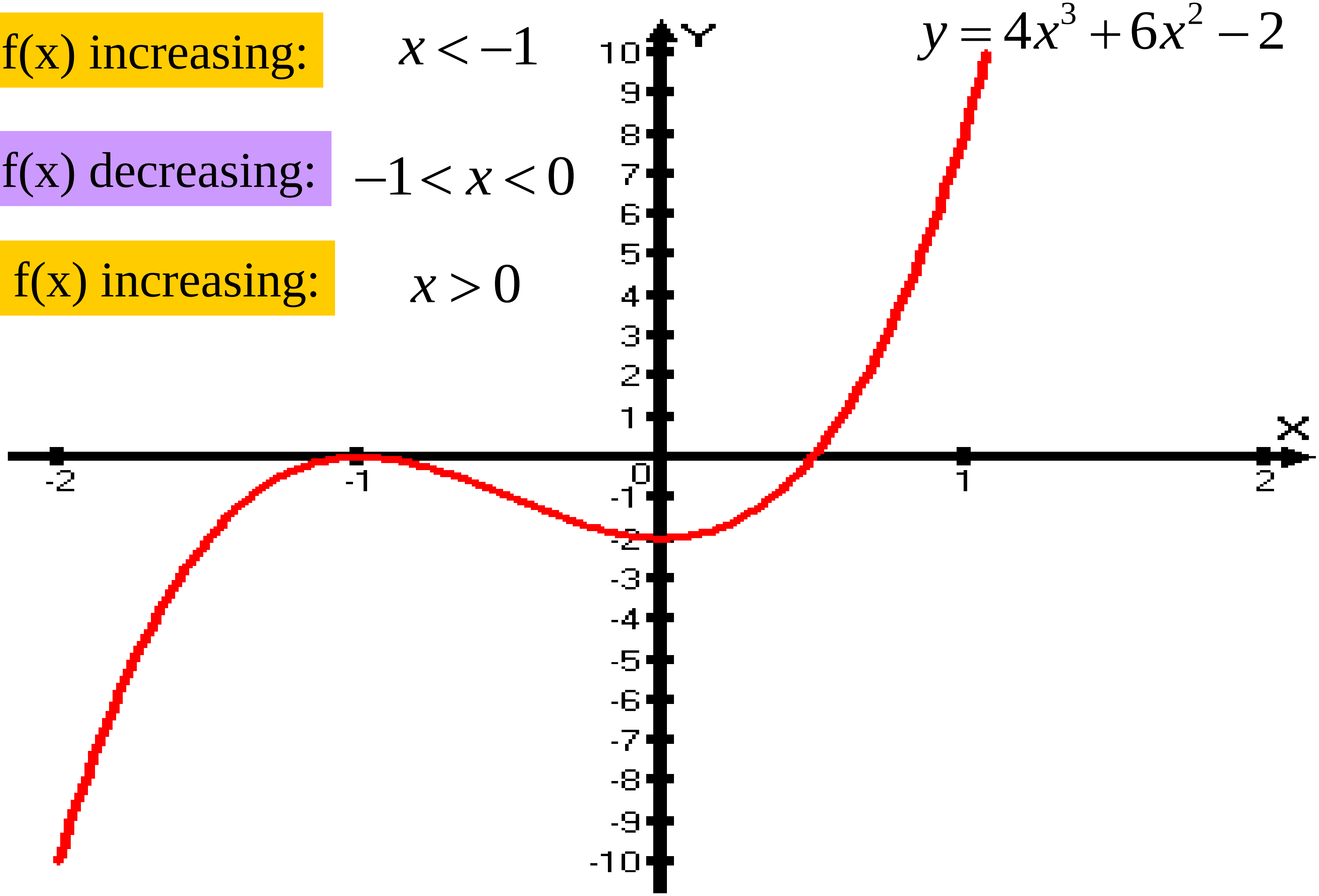
X	→	-1	→	0	→
$\frac{dy}{dx}$	+	0	-	0	+

The Proof

f(x) increasing: $x < -1$

f(x) decreasing: $-1 < x < 0$

f(x) increasing: $x > 0$



Example 3

Show that the function $y = \frac{x^3}{3} - 3x^2 + 9x + 4$ is never decreasing.

Solution:

1. Prepare for differentiation

$$y = \frac{x^3}{3} - 3x^2 + 9x + 4$$

$$y = \tfrac{1}{3}x^3 - 3x^2 + 9x + 4$$

2. Find dy / dx

$$\frac{dy}{dx} = x^2 - 6x + 9$$

3. Factorise (all terms involving x must have even powers).

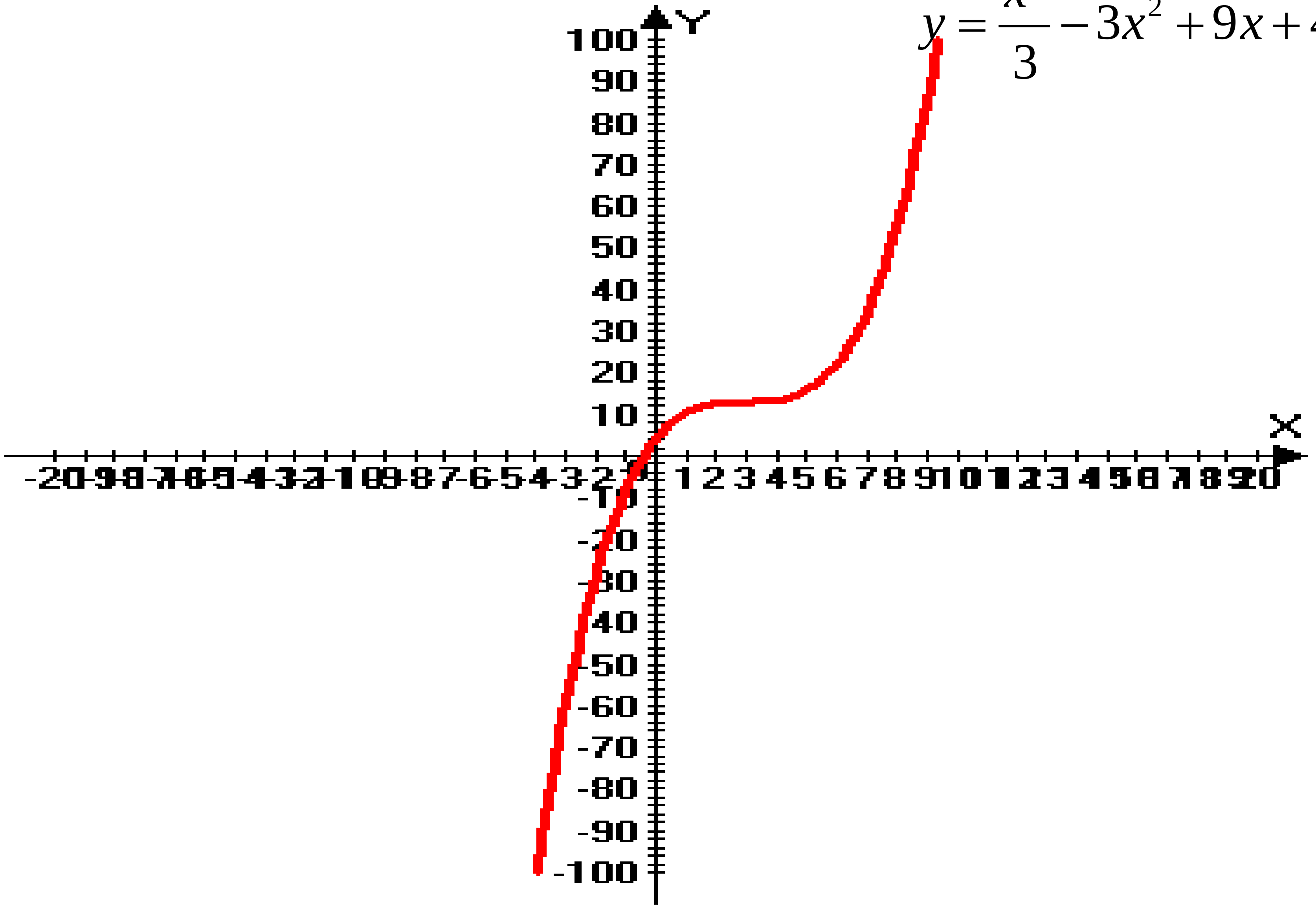
$$\frac{dy}{dx} = (x-3)^2$$

4. Make statement

Since $(x - 3)^2$ will always produce a positive gradient function is never decreasing.

<u>**The Proof**</u>

$$y = \frac{x^3}{3} - 3x^2 + 9x + 4$$



Heinemann , p.104, EX 6L, Q1 to 6