7.

Using the discriminant to find missing coefficients

$b^2 - 4ac$

A reminder

If $b^2 - 4ac = 0$: we get one (repeated) real root

If $b^2 - 4ac$ is positive: we get two distinct real roots

If $b^2 - 4ac$ is negative: there are no real roots

Sometimes in an exam or other situations we will be given some information and asked to find missing parts of the puzzle.

For instance, we may be told that the equation $x^2 - px + 8$ has two distinct real roots and asked to find the value of p.

In this case we would use what we already know:

we get two distinct real roots If $b^2 - 4ac$ is positive

Example 1

Find the values of p which make the roots of $9x^2 - px + 1$ equal

Solution:

a = 9, b = -p, c = 1

For equal roots: $b^2 - 4ac = 0$

$$\Rightarrow (-p)^2 - 4(9)(1) = 0$$

$$\Rightarrow p^2 - 36 = 0$$

$$\Rightarrow p^2 = 36$$

$$\Rightarrow \boxed{p = \pm 6}$$

Heinemann, p.152, EX 8I,
Q1(a), (b) & (c)

Example 2

Find the values of p which make the roots of

$x(x-3)+3=3x-3p$ real

Solution:

1. Must have the equation in the form $ax^2 + bx + c = 0$

2. Write down values of a, b and c

$$x(x-3)+3=3x-3p$$

$$x^2-3x+3=3x-3p$$

$$x^2-3x-3x+3+3p=0$$

$$x^2-6x+3+3p=0$$

a = 1, b = -6, c = (3+3p)

Example 2

Find the values of p which make the roots of

$x(x-3)+3=3x-3p$ real

a = 1, b = -6, c = (3+3p)

Solution:

3. Write down required condition

For real roots: $b^2-4ac\geq 0$

4. Sub values of a, b and c and solve equation / inequation

$$(-6)^2-4(1)(3+3p)\geq 0$$

$$36-4(3+3p)\geq 0$$

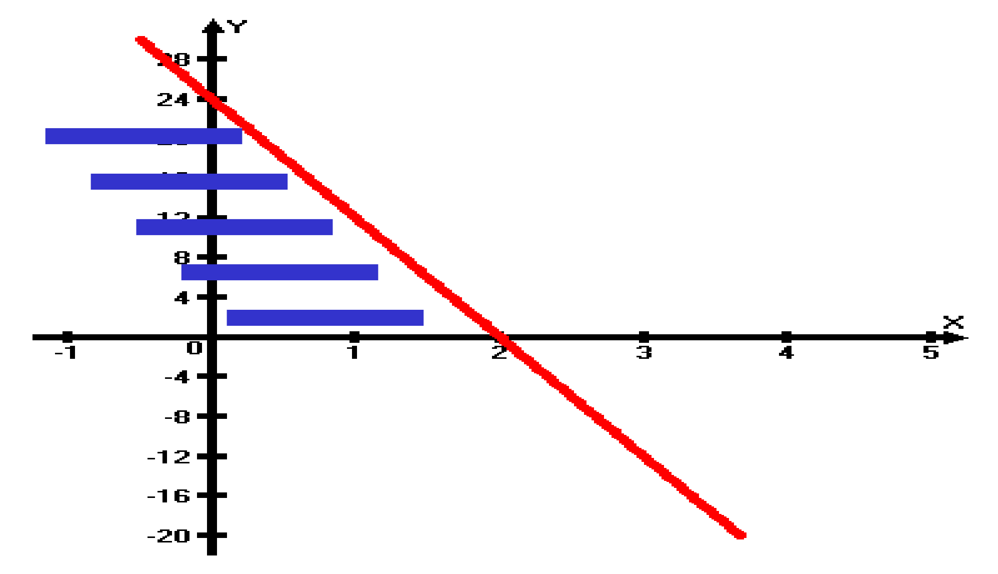
$$36-12-12p\geq 0$$

$$24-12p\geq 0$$

Careful!!

$$12p\leq 24$$

$$p\leq 2$$



Heinemann, p.152, EX 8I,
Q2

Example 3

Show that the roots of $x(kx+2)+2k = x(2x+3k)$ are always real

Solution:

$$x(kx+2)+2k = x(2x+3k)$$

1. Must have the equation in the form $ax^2 + bx + c = 0$

$$kx^2 + 2x + 2k = 2x^2 + 3kx$$

$$kx^2 - 2x^2 + 2x - 3kx + 2k = 0$$

$$(k-2)x^2 + (2-3k)x + 2k = 0$$

2. Write down values of a, b and c

a = (k-2), b =(2-3k), c = 2k

3. Find discriminant

$$b^2 - 4ac$$

$$= (2-3k)^2 - 4(k-2)(2k)$$

$$= 4 - 12k + 9k^2 - 8k^2 + 16k$$

Example 3

Show that the roots of $x(kx+2)+2k = x(2x+3k)$ are always real

Solution:

$$= 4 - 12k + 9k^2 - 8k^2 + 16k$$

$$= 4 + 4k + k^2$$

$$= (k+2)^2 + 4 - 4$$

$$= (k+2)^2$$

4. Factorise or complete square to produce a square

5. Make statement referring to fact that squares must always be +ve

As $(k+2)^2$ can never be –ve the discriminant is always positive or zero and so roots are always real.

Heinemann, p.152, EX 8I, Q5