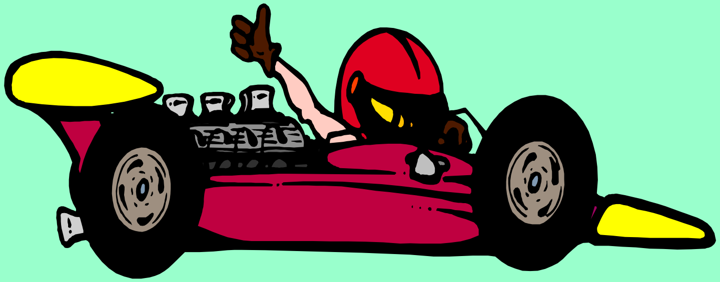
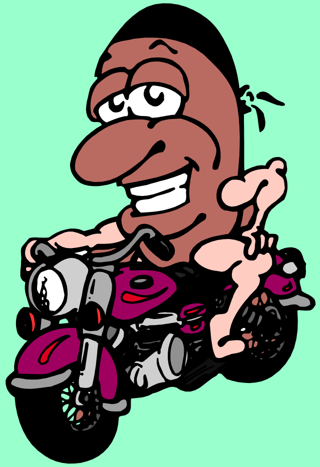
7. Differential Equations

An equation containing **derivatives** of a function





Differential Equations

- Differential equations are equations containing a derivative.
- They can be solved by integration to obtain a **general solution** with + C.
- To obtain a **specific solution** requires **additional information** .

Example 1

Find the solution of the differential equation $\dfrac{ds}{dt} = 4t + 1$

if s = 3 when t = 2.

Solution:

1. Integrate to find the anti-derivative

$$\frac{ds}{dt} = 4t + 1$$

$$s = \int (4t + 1)\ dt$$

$$s = \frac{4t^2}{2} + t + C$$

General Solution → $s = 2t^2 + t + C$

Example 1

Find the solution of the differential equation $\frac{ds}{dt} = 4t + 1$

if s = 3 when t = 2.

Solution:

$$s = 2t^2 + t + C$$

2. Use the extra information in the question to find the value of C.

IF s=3, t=2:

$$3 = 2(2)^2 + (2) + C$$

$$3 = 10 + C$$

$$C = -7$$

3. State **specific solution**

$$s = 2t^2 + t - 7$$

Example 2

Find the equation of the curve for which $\frac{dy}{dx} = 6x + \frac{5}{x^2}$

and which passes through the point (1 , 6).

Solution:

1. Integrate to find the anti-derivative

$$y = \int 6x + \frac{5}{x^2}\ dx$$

$$y = \int (6x + 5x^{-2})\ dx$$

$$y = \frac{6x^2}{2} + \frac{5x^{-1}}{-1} + C$$

$$y = 3x^2 - 5x^{-1} + C$$

General Solution

$$y = 3x^2 - \frac{5}{x} + C$$

Example 2

Find the equation of the curve for which $\frac{dy}{dx} = 6x + \frac{5}{x^2}$

and which passes through the point (1 , 6).

Solution:

$$y = 3x^2 - \frac{5}{x} + C$$

2. Use the extra information in the question to find the value of C.

IF x=1, y=6:

$$6 = 3(1)^2 - \frac{5}{1} + C$$

$$6 = -2 + C$$

$$C = 8$$

3. State **specific solution**

$$y = 3x^2 - \frac{5}{x} + 8$$

Heinemann, p.176, EX 9Q,
Q2 , 3 & 4