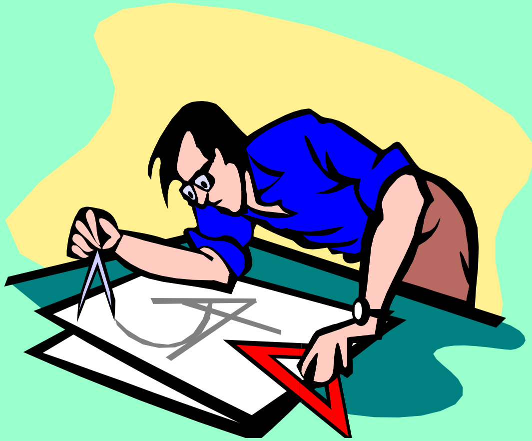
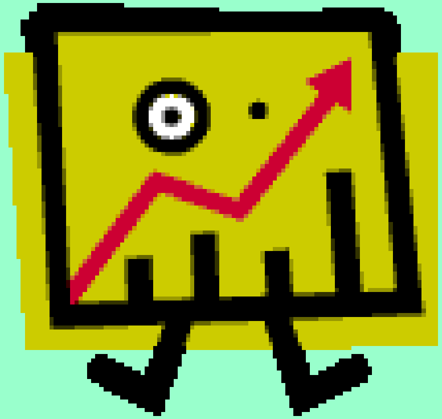
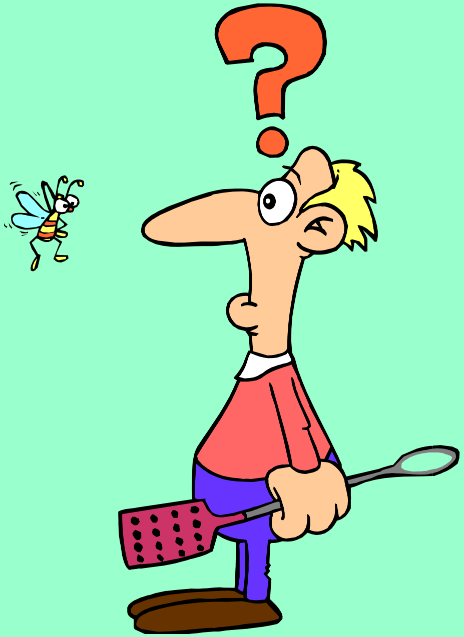
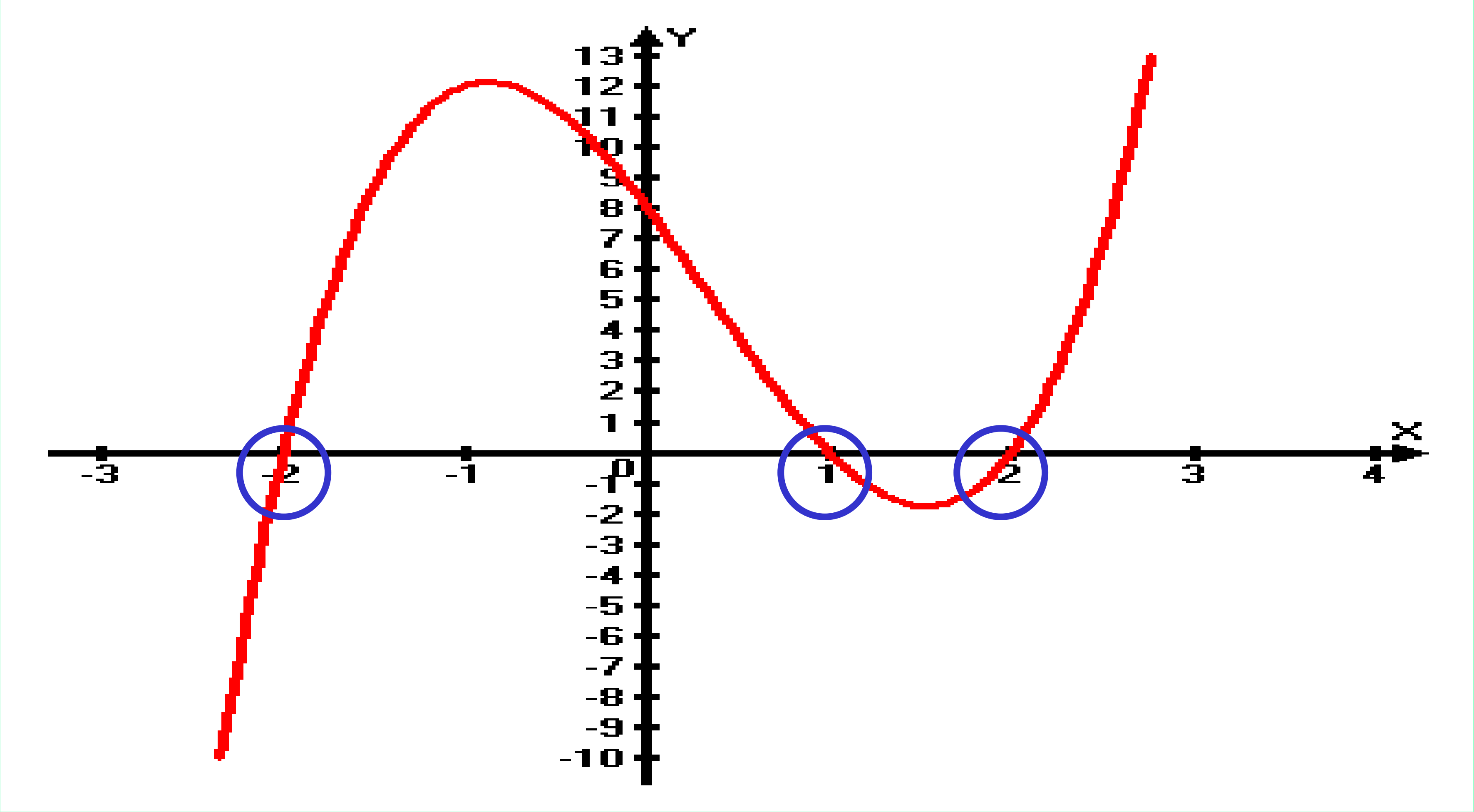




6. Polynomials and their graphs



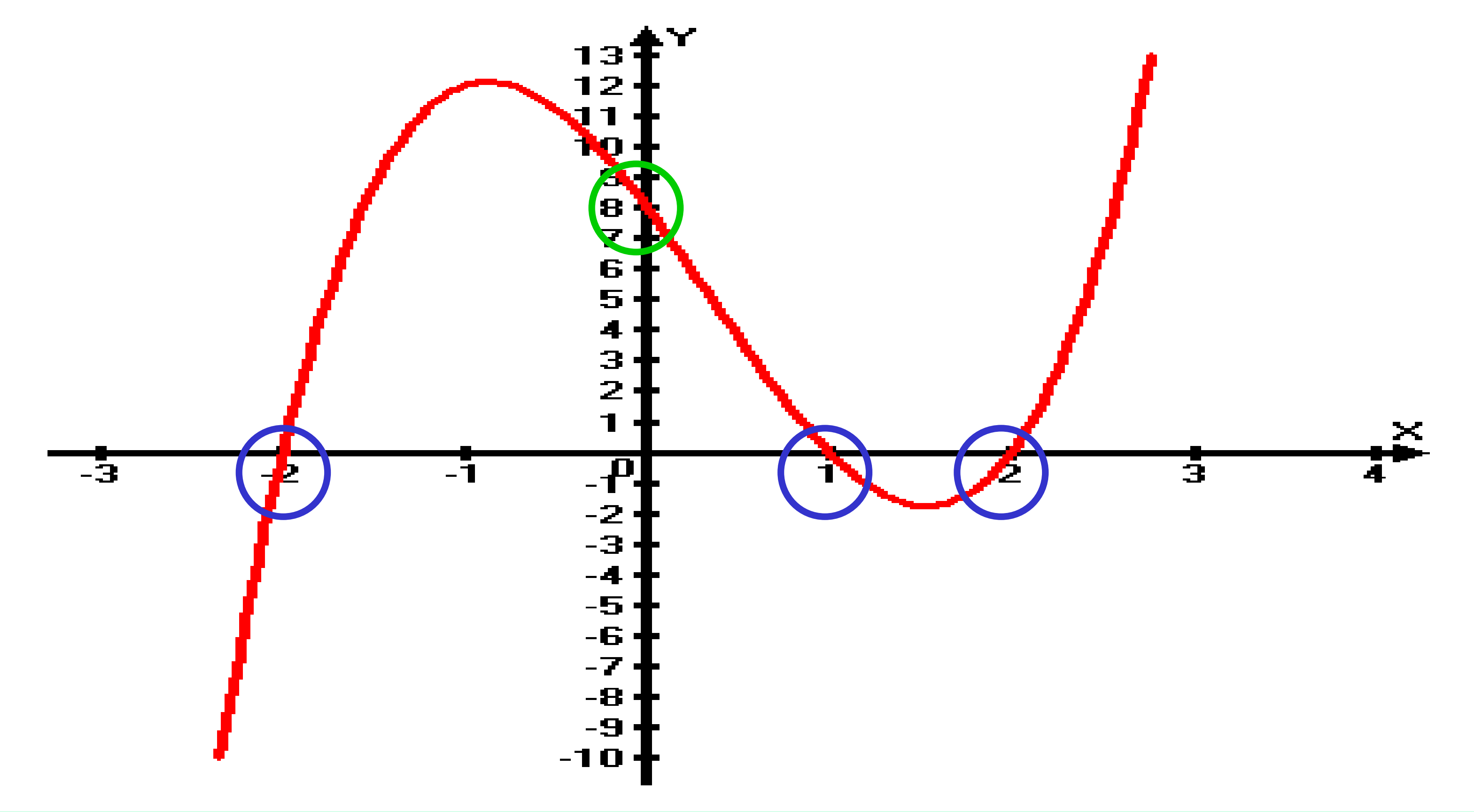
Finding the equation by inspecting the graph



Here is a graph of a cubic. If asked to find its equation how would we go about it?

For a cubic : $y = k(x - a)(x - b)(x - c)$, where a, b and c are roots.

Finding the equation by inspecting the graph



So in this case we have: $y = k\,(x + 2)(x - 1)(x - 2)$

At y intercept $x = 0$: $y = k \times 2 \times -1 \times -2$ → $8 = 4k$

$k = 2$

Finding the equation by inspecting the graph

Equation of curve: $y = k(x + 2)(x - 1)(x - 2)$ → $k = 2$

$y = 2(x + 2)(x - 1)(x - 2)$

$$y = (2x + 4)(x^2 - 2x - x + 2)$$

$$y = (2x + 4)(x^2 - 3x + 2)$$

$$y = 2x^3 - 6x^2 + 4x + 4x^2 - 12x + 8$$

$$y = 2x^3 - 2x^2 - 8x + 8$$

Heinemann, p.135, EX 7H,
Q1, 2, 3, 5, 6, 7

Sketching the curve of a cubic

Recall that when we want to sketch a graph we consider 5 things:

1. y-intercept $(x = 0)$
2. Roots $(y = 0)$
3. Stationary Points $dy/dx = 0$
4. Behaviour for large +ve x's
5. Behaviour for large –ve x's

Example 1

Sketch the graph of $y = x^3 - 3x - 2$

Solution:

1. y-intercept (x = 0) → $y = -2$ → (0,-2)

2. Roots (y = 0) $x^3 - 3x - 2 = 0$

Factors of -2 : $\pm 1, \pm 2$

$f(1) = 1 - 3 - 2 = -4$

$f(-1) = -1 + 3 - 2 = 0$ Success!!

So (x+1) is a factor and we can use x = -1 in synthetic division.

Example 1

Sketch the graph of $y = x^3 - 3x - 2$

Solution:

$x + 1 = 0$

$x = -1$

	x^3	x^2	x	x^0
-1	1	0	-3	-2
		-1	1	2
	1	-1	-2	0

$$f(x) = (x+1)(x^2 - x - 2)$$

$$f(x) = (x+1)(x+1)(x-2)$$

$(x + 1) = 0$ $\quad$ $(x - 2) = 0$

$x = -1$ $\quad$ $x = 2$

Points are: (-1,0) $\quad$ (2,0)

Example 1

Sketch the graph of $y = x^3 - 3x - 2$

Solution:

3. Stationary Points dy/dx = 0

$$\frac{dy}{dx} = 3x^2 - 3$$

At SP's dy/dx = 0

$$3x^2 - 3 = 0$$

$$3(x^2 - 1) = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

For x = 1, y = -4 → (1 , -4)

For x = -1, y = 0 → (-1 , 0)

Example 1

Sketch the graph of y = x^3 - 3x - 2

Solution:

$$\frac{dy}{dx} = 3x^2 - 3 \qquad x = \pm 1$$

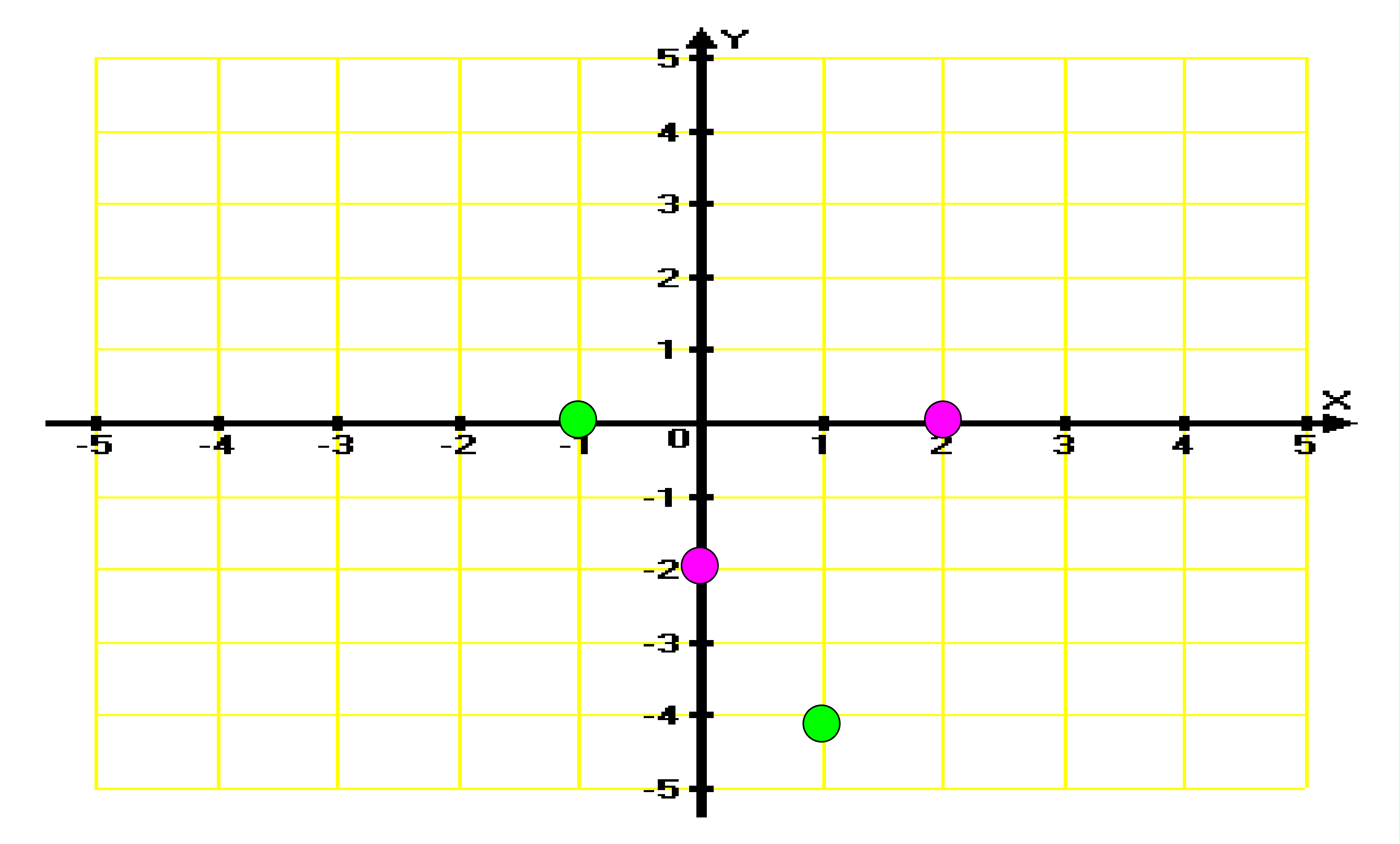
X	-1^{-}	-1	→	1	1^{+}
$\frac{dy}{dx}$	+ve	0	-ve	0	+ve
Slope	/	—	\	_	/

Maximum tp at (-1 , 0)

Minimum tp at (1 , -4)

4. Behaviour for large +ve x's → +ve

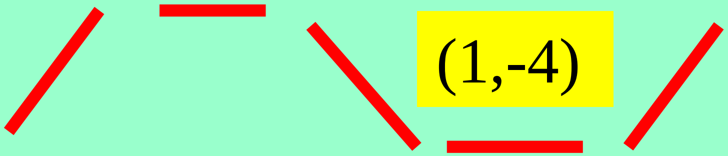
5. Behaviour for large –ve x's → -ve



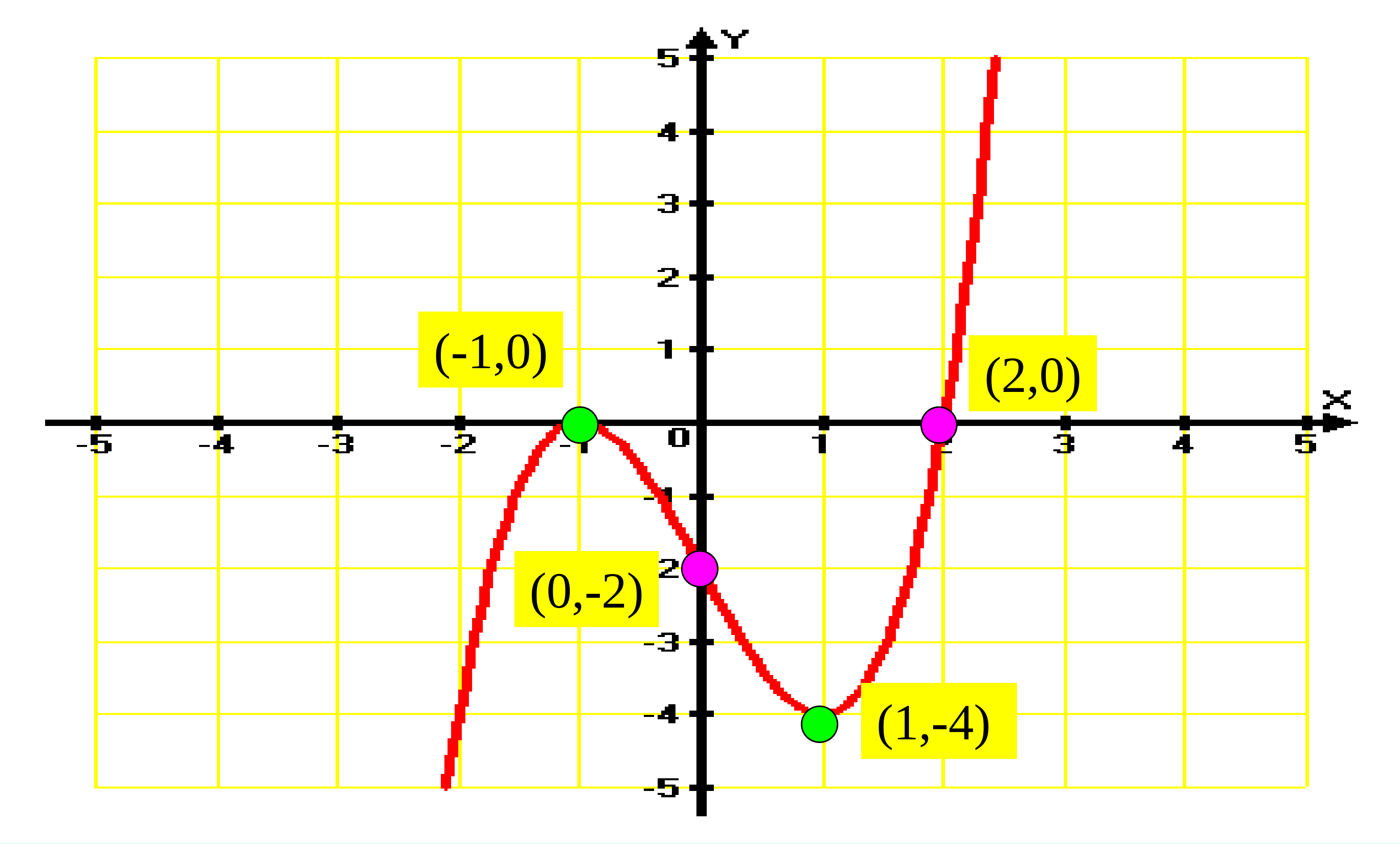
1. y-intercept (0,-2)

2. Roots (-1,0) (2,0)

3. Stationary Points (-1,0)



(1,-4)



Annotate your graphs!!!!

Heinemann, p.137, EX 7I,
Q7 & 8