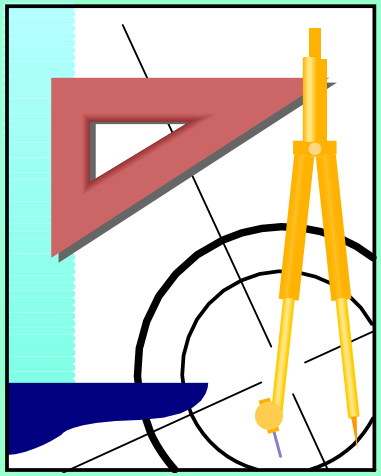
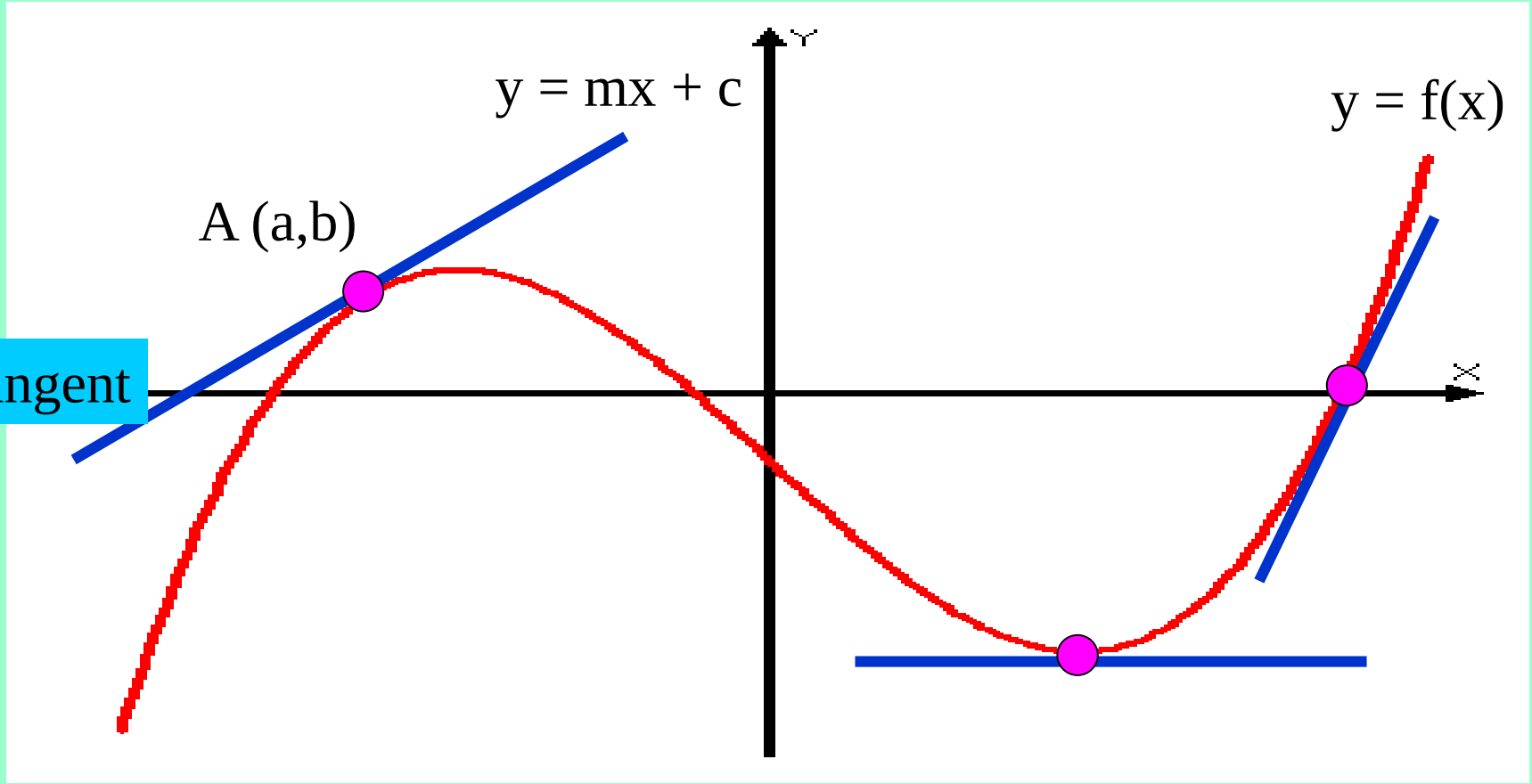


6. The Equation of the tangent

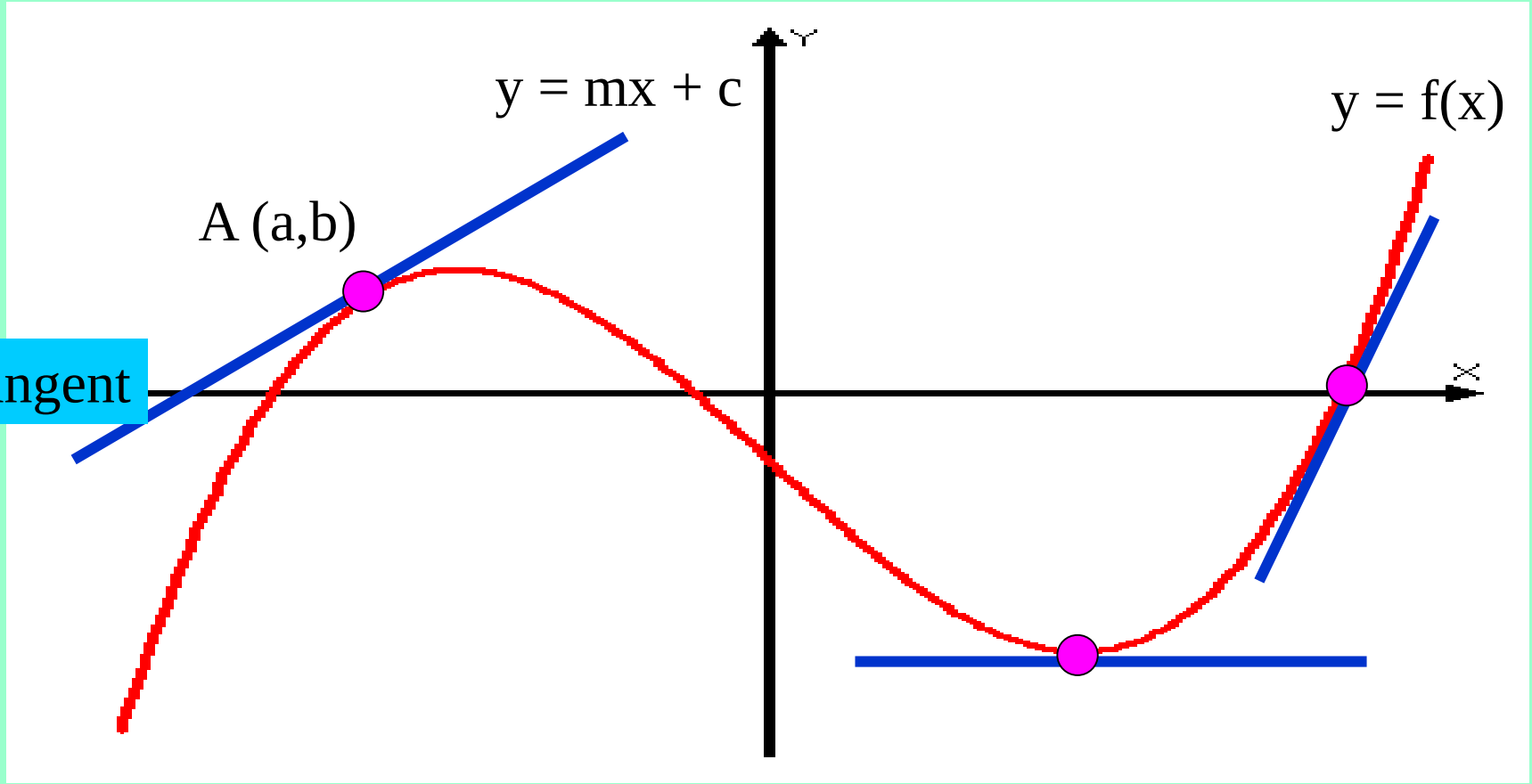
The Tangent to the Graph



So far we have differentiated functions, stating that the derived function provides us with the gradient of the tangent to the graph.

We have seen that whilst the derived function is the same throughout the curve the actual gradient is dependant on the x-coordinate.

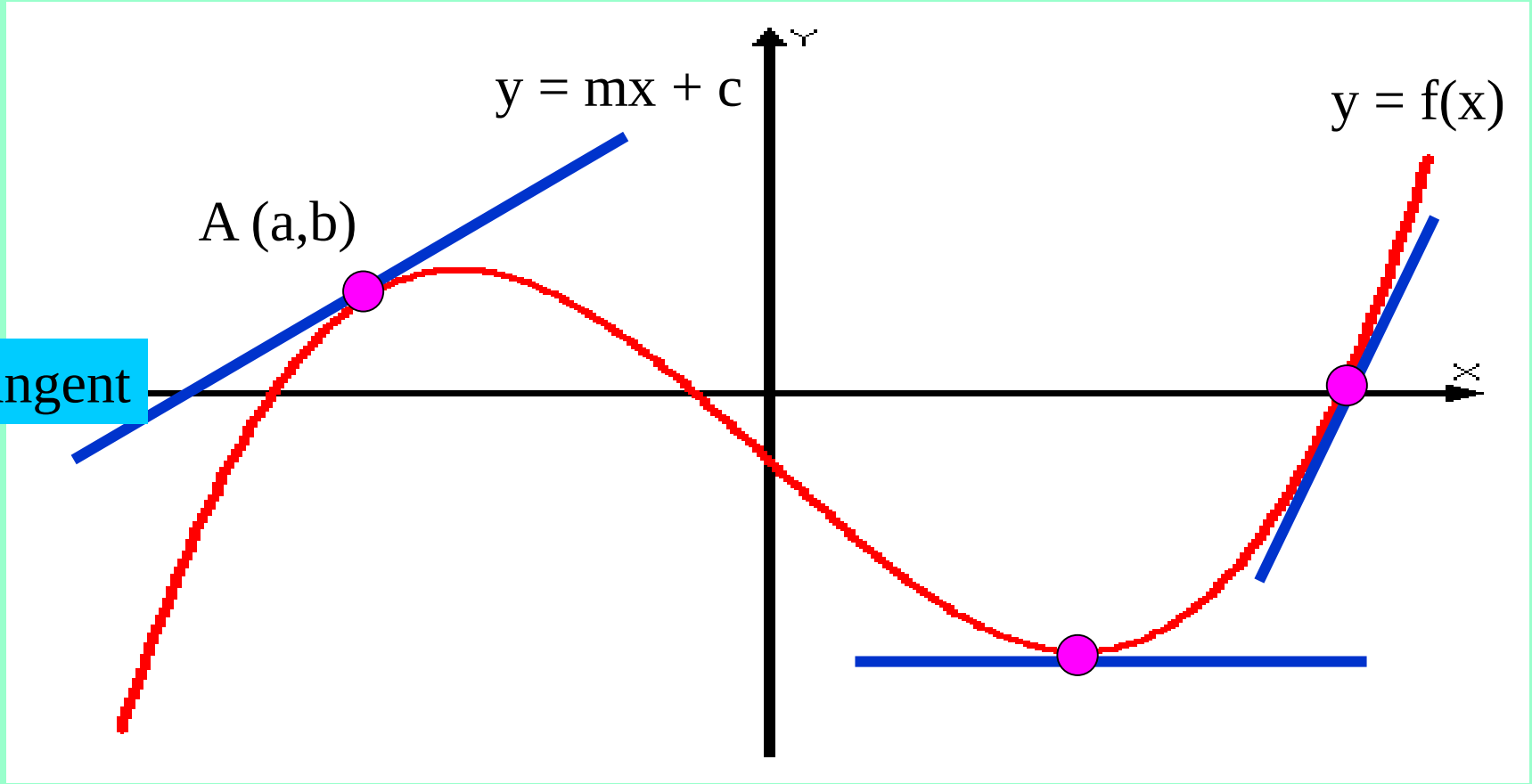
The Tangent to the Graph



Now we want to find the equation of the tangent.

The tangent is a straight line. We know from previous work on the straight line that y - b= m(x-a). How can we find m?

The Tangent to the Graph



At any point gradient of curve = gradient of tangent

As the gradient of a straight line is the same throughout the line if we can find the gradient of the curve at A this is the gradient we can use for the equation of the tangent. So $m = f'(a)$

Finding the Equation of the Tangent

Copy the following:

To find the equation of the tangent we:

1. Must have both coordinates of the point on the curve

2. Must have the gradient at that point.

3. Substitute these details into: $y - b = m(x - a)$

Example 1

NAB

Find the equation of the tangent to the curve $y = 5x^4 + 1$ at x = 1

Solution:

1. If not given, find y-coordinate

2. Find dy / dx

3. Find gradient at x-coordinate

4. Substitute values into $y - y_1 = m(x - x_1)$

$$y = 5x^4 + 1$$

$$y = 5(1)^4 + 1 \quad = 6$$

$$\frac{dy}{dx} = 20x^3$$

$$\frac{dy}{dx}(1) = 20(1)^3 \quad = 20$$

Equation of tangent at (1,6):

$$y - 6 = 20(x - 1)$$

$$y - 6 = 20x - 20$$

$$y = 20x - 14$$

Example 2

Show that there is only one tangent to the curve $y = 5x^2 + 6x$ with gradient 36

Solution:

1. Note we are being asked to prove that dy/dx = 36 has <u>only one solution.</u>

2. Find dy/dx

$$\frac{dy}{dx} = 10x + 6$$

3. Make an equation with given gradient and solve

$$36 = 10x + 6$$

$$10x = 30$$

$$x = 3$$

4. Make statement

As dy/dx = 36 has only one solution there is only one tangent with that gradient.

Example 3

Find the point of contact of the curve $y = x^2 + 3x - 8$ at which the gradient is 9.

Solution:

1. Find dy / dx

$$y = x^2 + 3x - 8$$

$$\frac{dy}{dx} = 2x + 3$$

2. Make dy / dx equal to given gradient and solve.

$$2x + 3 = 9$$

$$2x = 6$$

$$x = 3$$

3. Substitute into **original equation** to find y-coordinate.
(DO NOT USE dy / dx.)

y-coord when x = 3:

$$y = (3)^2 + 3(3) - 8$$

$$y = 9 + 9 - 8 = 10$$

4. Make statement

Point of contact (3 , 10)

Heinemann , p.101, EX 6J,
Q1, 3, 5, 6 & 7