6.

Using the discriminant to find nature of roots

$b^2 - 4ac$

What is the discriminant?

Recall the quadratic formula for finding roots:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

→ Discriminant (Δ)

The discriminant is the $b^2 - 4ac$ part.
This is because it determines the number of roots of the quadratic and their nature.

If $b^2 - 4ac = 0$:

$$x = \frac{-b \pm \sqrt{0}}{2a} = -\frac{b}{2a}$$

we get one (repeated) root

If $b^2 - 4ac$ is positive:

$$x = \frac{-b + \sqrt{\Delta}}{2a}$$

or

$$x = \frac{-b - \sqrt{\Delta}}{2a}$$

What is the discriminant?

If $b^2 - 4ac$ is negative:

$$x = \frac{-b \pm \sqrt{-\Delta}}{2a}$$

This would mean taking the square root of a negative which we do not know how to do.

Thus, if the discriminant is negative there are no real roots.

In summary:

If $b^2 - 4ac = 0$: we get one (repeated) real root

If $b^2 - 4ac$ is positive: we get two distinct real roots

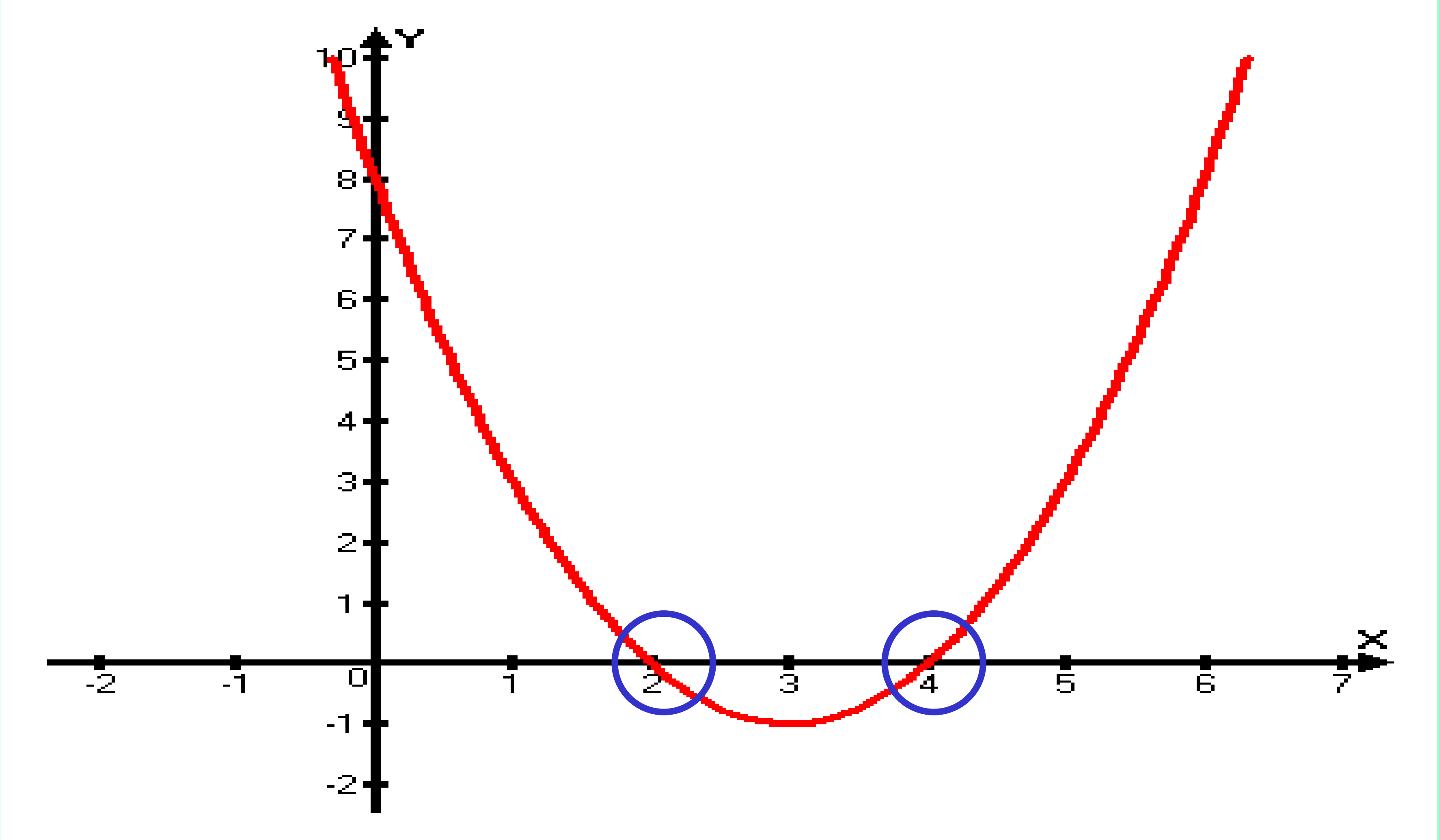
If $b^2 - 4ac$ is negative: there are no real roots

Take the equation $x^2 - 6x + 8 = 0$

$$b^2 - 4ac = (-6)^2 - 4(1)(8)$$ $= 4$

If $b^2 - 4ac$ is positive:

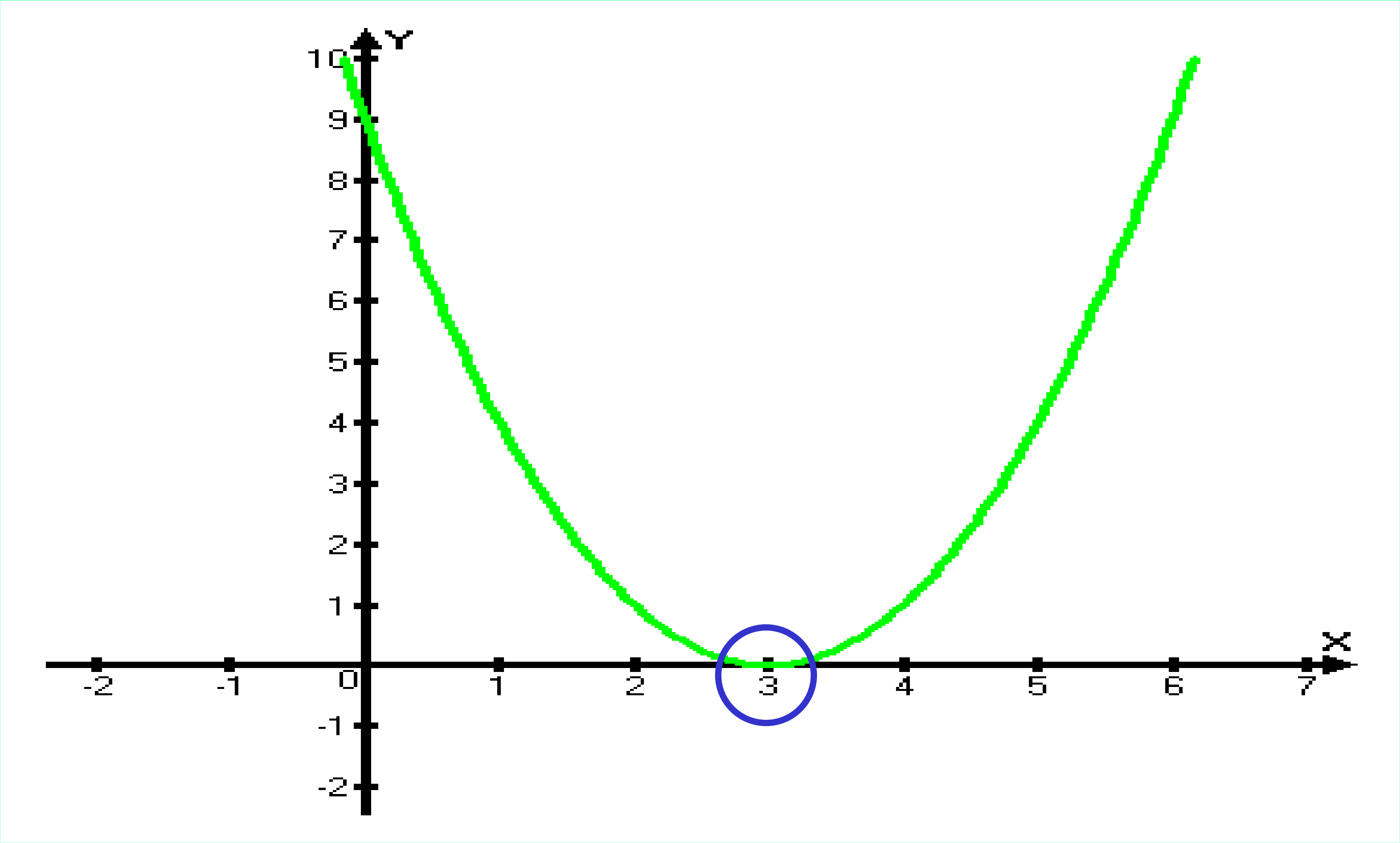
we get two distinct real roots



Take the equation $x^2 - 6x + 9 = 0$

$$b^2 - 4ac = (-6)^2 - 4(1)(9)$$ $= 0$

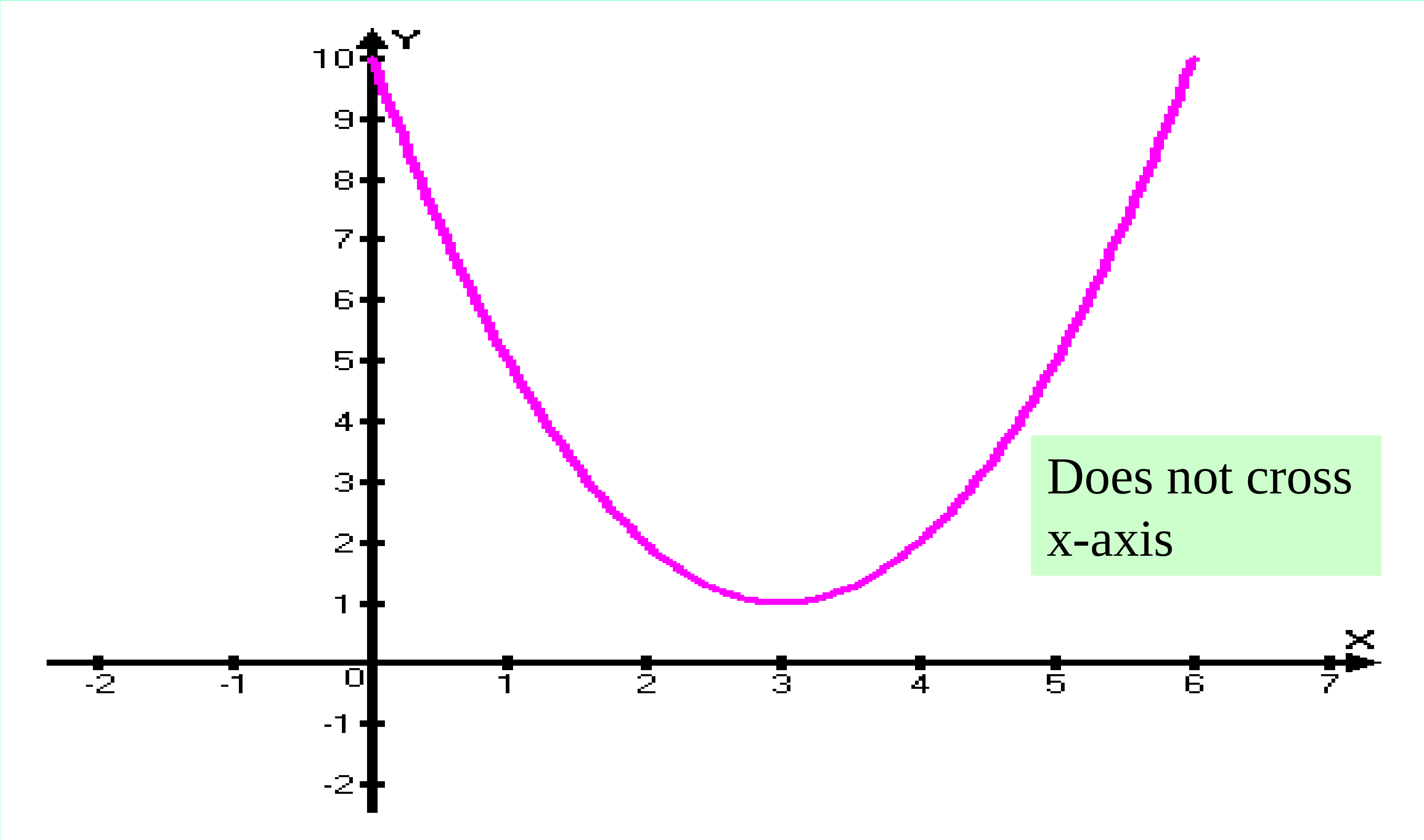
If $b^2 - 4ac = 0$: we get one (repeated) real root



Take the equation $x^2 - 6x + 10 = 0$

$$b^2 - 4ac = (-6)^2 - 4(1)(10)$$ = - 4

If $b^2 - 4ac$ is negative: there are no real roots



Does not cross x-axis

Example 1

NAB

Determine the nature of the roots of $3x^2 - 2x + 7$

Solution:

a = 3, b = -2, c = 7

$$b^2 - 4ac$$

$$= (-2)^2 - 4(3)(7)$$

$$= 4 - 84$$

$$= -80$$

As $b^2 - 4ac$ is negative there are no real roots

Heinemann, p.151, EX 8H, Q1