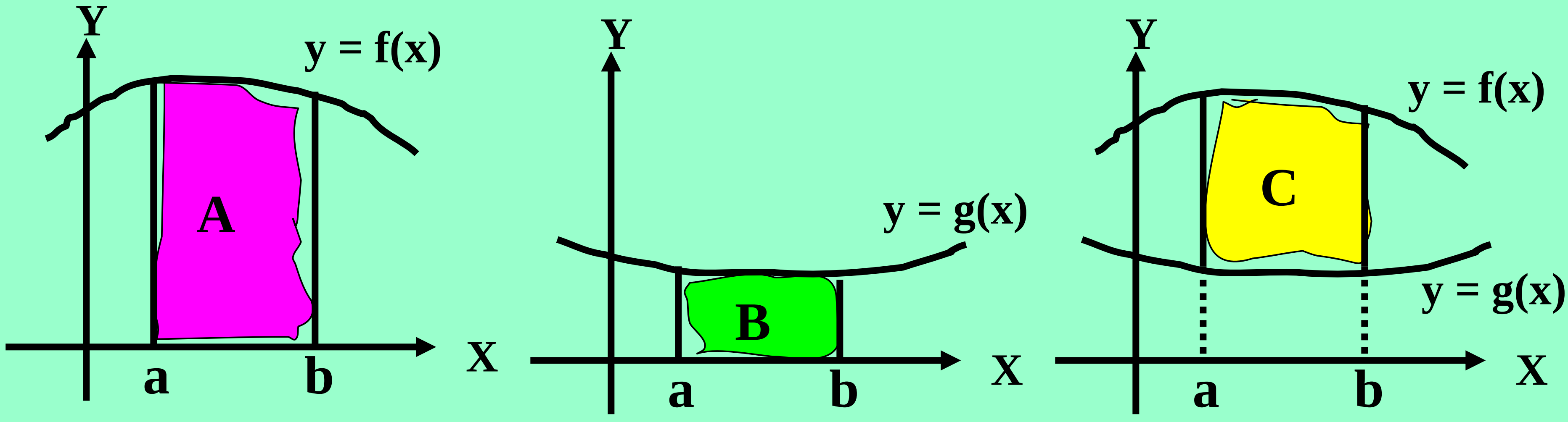
6. Area between two graphs

AREAS BETWEEN CURVES



$$Area\ \ A = \int_a^b f(x)dx$$

$$Area\ \ B = \int_a^b g(x)dx$$

Area C = **AreaA** - **AreaB**

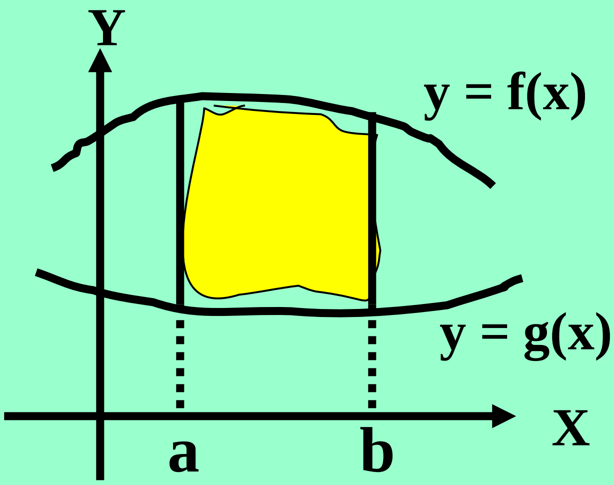
$$= \int_a^b f(x)dx \ - \int_a^b g(x)dx$$

$$= \int_a^b f(x) - g(x)\ dx$$

$$\int_a^b \text{upper curve} - \text{lower curve}$$

AREAS BETWEEN CURVES

Copy the following:



In the interval $a < x < b$ if $f(x) > g(x)$

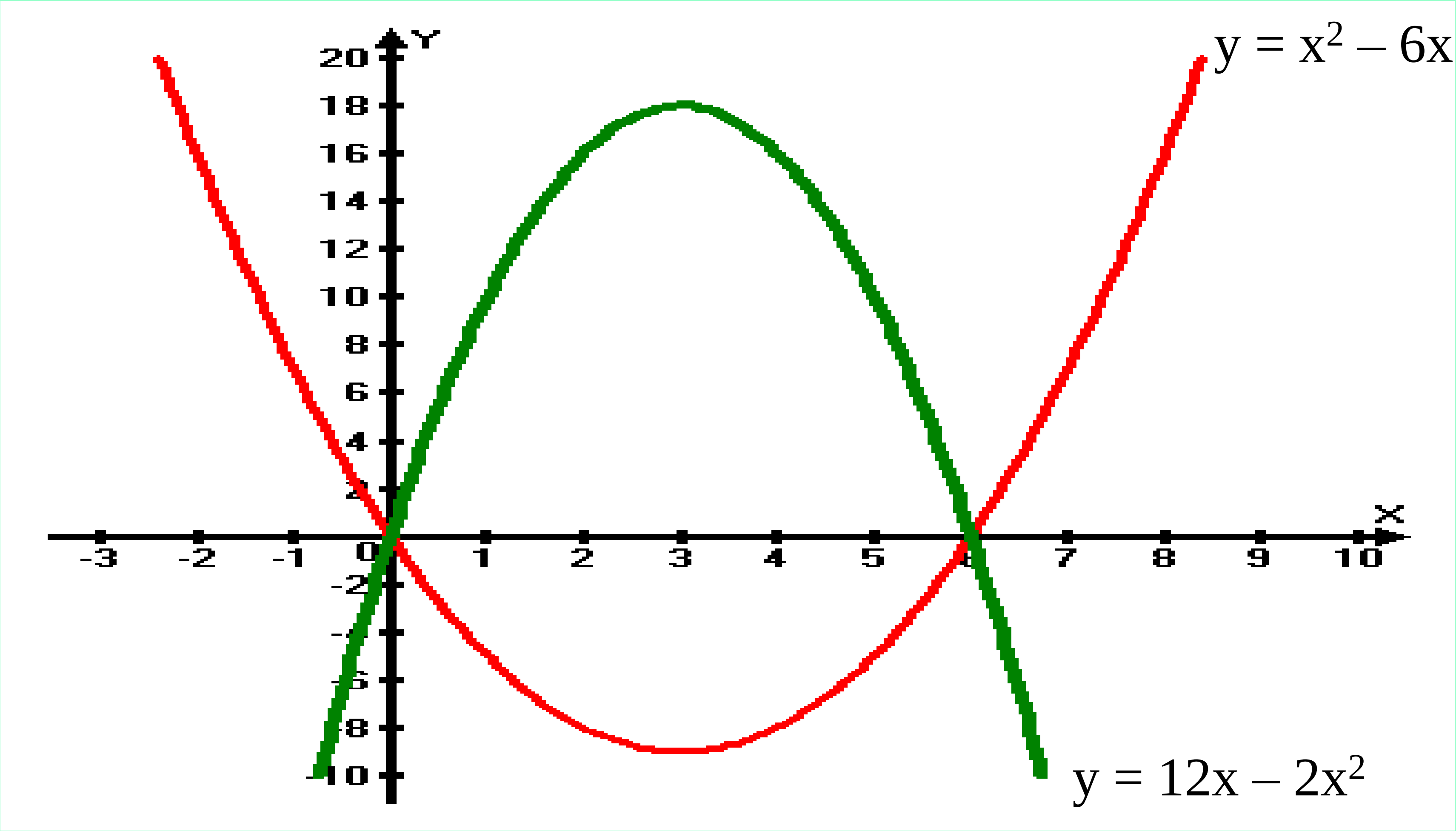
then the area enclosed between the curves

from $x = a$ to $x = b$ is given by

Area $= \int_a^b f(x) - g(x) \; dx$

$\int_a^b$ upper curve – lower curve

Example 1



Find the area enclosed by the parabolas $y = x^2 - 6x$ and $y = 12x - 2x^2$

Example 1

Find the area enclosed by the parabolas $y = x^2 - 6x$ and

$y = 12x - 2x^2$

Solution:

1. Find limits by setting f(x) = 0 and factorising

From graph we see limits are 0 and 6

2. Prepare and then evaluate integral between these limits

$$Area = \int_a^b f(x) - g(x)\ dx \longrightarrow \int_a^b \text{upper curve} - \text{lower curve}$$

$$Area = \int_0^6 12x - 2x^2 - (x^2 - 6x)\ dx \longleftarrow$$

Must put 2^{nd} function in brackets

Example 1

Find the area enclosed by the parabolas $y = x^2 - 6x$ and

$y = 12x - 2x^2$

Solution:

$$Area = \int_0^6 12x - 2x^2 - (x^2 - 6x)\ dx$$

$$Area = \int_0^6 12x - 2x^2 - x^2 + 6x\ dx$$

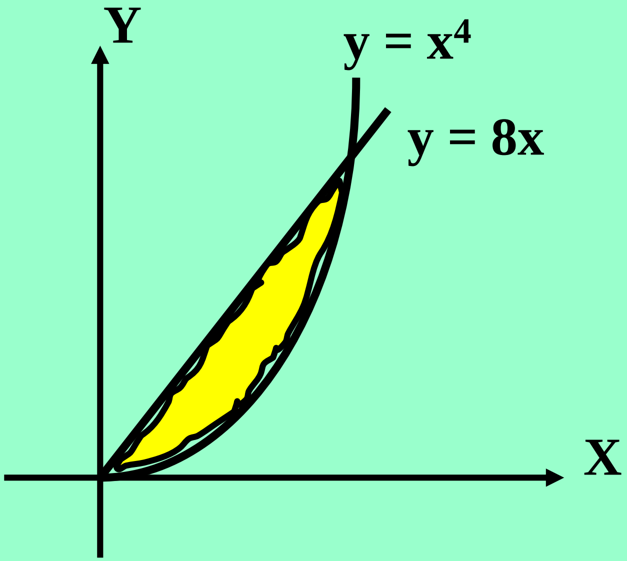
$$Area = \int_0^6 18x - 3x^2\ dx$$

$$Area = \left[9x^2 - x^3\right]_0^6$$

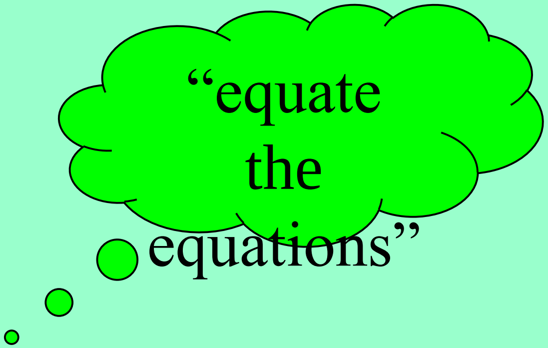
$Area = (324 - 216) - 0$ → **$Area$ = 108 units^2**

Example 2

NAB



OK! Who hid my banana?



Find shaded area!

1. Find limits by setting f(x) = 0 and factorising

Limits:

$x^4 = 8x$

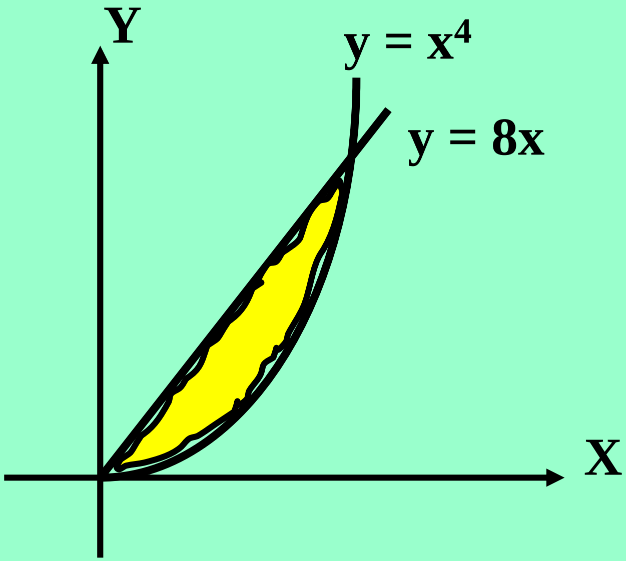
$x^4 - 8x = 0$

$x(x^3 – 8) = 0$

$x = 0$ or $x^3 = 8$

$x = 2$

Example 2



OK! Who hid my banana?

Find shaded area!

2. Prepare and then evaluate integral between these limits

$$Area = \int_a^b f(x) - g(x)\ dx$$

$$Area = \int_0^2 8x - (x^4)\ dx$$

$$= \left[4x^2 - {}^{1}/_{5}x^5\right]_0^2$$

$$= (16 - {}^{32}/_{5}) - 0$$

$$= 9\,{}^{3}/_{5} \text{ units}^2$$

Example 3

Find the finite area enclosed by the parabolic curves $y = 2x^2 + x – 9$ and $y = x^2 + 2x – 3$.

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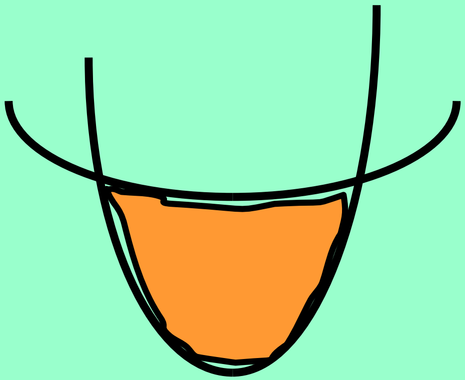
"equate the equations"

Roughly



1. Find limits by setting f(x) = 0 and factorising

Limits:

$$2x^2 + x – 9 = x^2 + 2x – 3$$

$$x^2 - x – 6 = 0$$

$$(x - 3)(x + 2) = 0$$

x = 3 **or** **x = -2**

Example 3

Find the finite area enclosed by the parabolic curves
$y = 2x^2 + x - 9$ and $y = x^2 + 2x - 3$.

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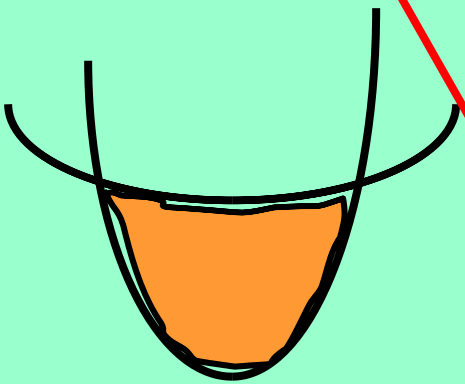
Roughly



2. Find which curve is upper curve by subbing ANY value between two limits into both functions.

<u>For $-2 < x < 3$</u>

taking $x = 0$

$2x^2 + x - 9 = -9$

& $x^2 + 2x - 3 = -3$

so $y = x^2 + 2x - 3$ is the upper curve

Example 3

Find the finite area enclosed by the parabolic curves
$y = 2x^2 + x - 9$ and $y = x^2 + 2x - 3$.

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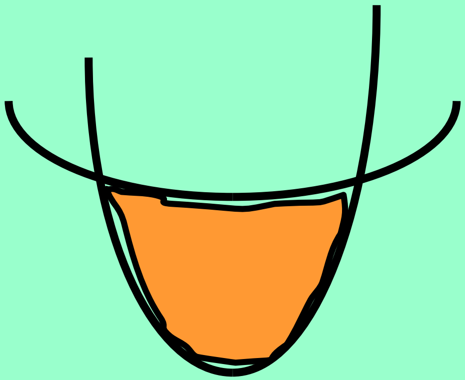
Roughly



3. Prepare and then evaluate integral between the limits

$$Area = \int_{-2}^{3} x^2 + 2x - 3 - (2x^2 + x - 9)\ dx$$

$$Area = \int_{-2}^{3} x^2 + 2x - 3 - 2x^2 - x + 9\ dx$$

$$Area = \int_{-2}^{3} -x^2 + x + 6\ dx$$

$$-\frac{x^3}{3} + \frac{x^2}{2} + 6x$$

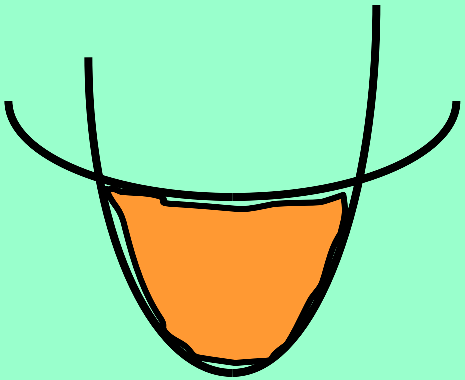
$$= \left[-\tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 + 6x \right]_{-2}^{3}$$

Example 3

Find the finite area enclosed by the parabolic curves $y = 2x^2 + x - 9$ and $y = x^2 + 2x - 3$.

****

Roughly



3. Prepare and then evaluate integral between the limits

$$= \left[-\tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 + 6x \right]_{-2}^{3}$$

$$= (-9 + 4\tfrac{1}{2} + 18) - (\tfrac{8}{3} + 2 - 12)$$

$$= 20\tfrac{5}{6} \text{ units}^2$$

$$-\frac{x^3}{3} + \frac{x^2}{2} + 6x$$

Heinemann, p.173, EX 9O, Q1(a) & (b)

p.174, EX 9P, Q1 and 2