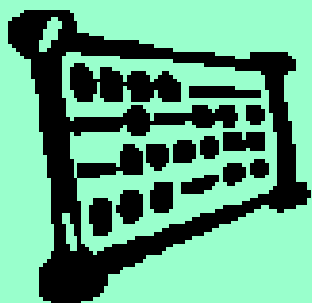


5. Solving Polynomial Equations



Example 1

Factorise $x^3 - 12x - 16$ fully and hence solve $x^3 - 12x - 16 = 0$

Solution :

First as this is a cubic it has form $k(x-a)(x-b)(x-c)$

the a, b and c parts will multiply together to produce the constant term

So we must look at the factors of 16: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

We want to find which of these factors make $f(x) = 0$

$$f(x) = x^3 - 12x - 16$$

$$f(1) = 1 - 12 - 16 \neq 0$$

$$f(-1) = -1 + 12 - 16 \neq 0$$

$$f(2) = 8 - 24 - 16 \neq 0$$

$$f(-2) = -8 + 24 - 16 = 0$$



Success : so $(x + 2)$ must be a factor

Example 1

Factorise $x^3 - 12x - 16$ fully and hence solve $x^3 - 12x - 16 = 0$

Solution :

Success : so $(x + 2)$ must be a factor

$x + 2 = 0$

$x = -2$

x^3	x^2	x	x^0
1	0	-12	-16
↓	-2	4	16
1	-2	-8	0

So if : $x^3 - 12x - 16 = 0$

$$(x + 2)(x^2 - 2x - 8) = 0$$

$$(x + 2)(x + 2)(x - 4) = 0$$

$$x + 2 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = -2$$

$$\text{or} \quad x = 4$$

Example 2 (powers greater than 3)

Find where the graph of $y = 2x^4 - 9x^3 + 6x^2 + 11x - 6$ cuts the x-axis

Solution : NB. Cuts the x-axis when $f(x) = 0$

First consider the factors of the constant term : $\pm 1, \pm 2, \pm 3, \pm 6$

$$f(x) = 2x^4 - 9x^3 + 6x^2 + 11x - 6$$

$$f(1) = 2 - 9 + 6 + 11 - 6 = 4 \neq 0$$

$$f(-1) = 2 + 9 + 6 - 11 - 6 = 0 \leftarrow \text{Success !!} \img alt="speaker icon" data-bbox="710 605 745 650"/>$$

Now use the value $x = -1$ to perform synthetic division

Example 2 (powers greater than 3)

Find where the graph of $y = 2x^4 - 9x^3 + 6x^2 + 11x - 6$ cuts the x-axis

Solution :

$x + 1 = 0$

• • • $x = -1$

x^4	x^3	x^2	x	x^0
2	-9	6	11	-6
↓	-2	11	-17	6
2	-11	17	-6	0

$$f(x) = (x+1)(2x^3 - 11x^2 + 17x - 6)$$

Must do it all again to factorise 2nd bracket

factors of the constant term : $\pm 1, \pm 2, \pm 3, \pm 6$

$$f(1) = 2 - 11 + 17 - 6 = 2$$

$$f(2) = 16 - 44 + 34 - 6 = 0$$

$$f(-1) = -2 - 11 - 17 - 6 = -36$$

Example 2 (powers greater than 3)

Find where the graph of $y = 2x^4 - 9x^3 + 6x^2 + 11x - 6$ cuts the x-axis

Solution:

$$x - 2 = 0$$

$$\bullet \bullet \bullet x = 2$$

x^3	x^2	x	x^0
2	-11	17	-6
↓	4	-14	6
2	-7	3	0

$$f(x) = (x+1)(x-2)(2x^2 - 7x + 3)$$

$$f(x) = (x+1)(x-2)(2x-1)(x-3)$$

Cuts the x-axis when $f(x) = 0$

$$(x + 1) = 0$$

$$(x - 2) = 0$$

$$(2x - 1) = 0$$

$$(x - 3) = 0$$

Points are:

$$(-1, 0)$$

$$(2, 0)$$

$$(0.5, 0)$$

$$(3, 0)$$

Heinemann, p.134, EX 7G,
Q1, 2 & 4