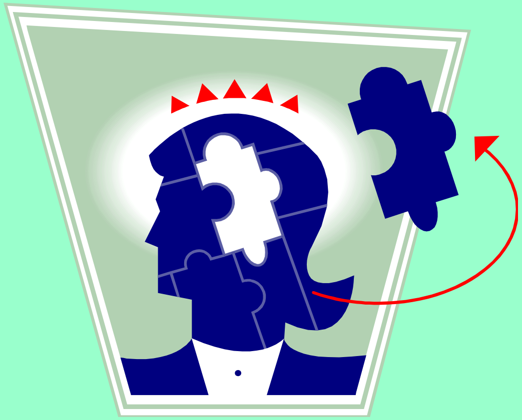
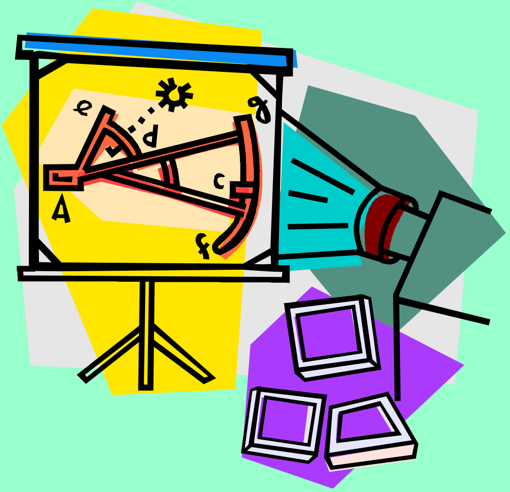


5.

Natural logs



Natural Logarithms

In the 1700's a Swiss mathematician, named Leonhard Euler, found that there was an exponential function which had the unusual property that the graph of its derivative was exactly the same as the graph of the original function.

This is known as the exponential function to base ***e***:

$$f(x) = 2.718^x = \exp(x) = e^x$$

The inverse of this function is the logarithm to base ***e*** :

$$\log_e x = \ln(x)$$

The natural logarithm

We use the fact that these functions are inverses to solve equations.

Example 1

Solve $\ln x = 1 \cdot 3$

Solution:

$$\ln x = 1 \cdot 3$$

Take exponential of each side

$$\exp(\ln x) = \exp(1 \cdot 3)$$

$$x = \exp(1 \cdot 3)$$

$$x = 3 \cdot 67$$

Look for one of these keys on calculator

e ^

e^x

Heinemann, p.289, EX 15H, Q1

This is not the end

Example 2

Solve $e^{(2x+2)} = 12$

Solution:

$$e^{(2x+2)} = 12$$

Take natural log of each side

$$(2x+2) = \ln(12)$$

$$2x = \ln(12) - 2$$

$$x = \frac{\ln(12) - 2}{2}$$

$$x = 0.24$$

Look for one of these keys on calculator

ln **$\log_e$**

Heinemann, p.289, EX 15H, Q2

This is not the end

Example 3

Solve $5^{(2x)} = 35$

Solution:

Take natural log of each side

$$5^{(2x)} = 35$$

Take powers to front

$$\ln 5^{2x} = \ln 35$$

$$2x \ln 5 = \ln 35$$

$$2x = \frac{\ln 35}{\ln 5}$$

$$2x = 2.209...$$

$$x = 1 \cdot 10$$

Heinemann, p.289, EX 15H, Q3(a) to (d)

This is not the end

Example 4

The amount of a radioactive substance is given by $A = A_0 e^{-kt}$.

In 3 minutes 10 grams of Bismuth reduce to 9 grams through radioactive decay.

(a) Find k

(b) Find the half-life of bismuth.

Solution to (a):

$$A = A_0 e^{-kt}$$

$$A_0 = 10$$

$$A = 9$$

$$t = 3$$

$$9 = 10\exp(-k \times 3)$$

$$10\exp(-3k) = 9$$

$$\exp(-3k) = \tfrac{9}{10} = 0.9$$

$$(-3k) = \ln\left(0.9\right)$$

Take natural log of each side

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In 3 minutes 10 grams of Bismuth reduce to 9 grams through radioactive decay.

(a) Find k

(b) Find the half-life of bismuth.

Solution to (a):

$$A = A_0 e^{-kt}$$

$A_0 = 10$

$A = 9$

$t = 3$

$$(-3k) = \ln(0.9)$$

$$k = \frac{\ln(0.9)}{-3}$$

$$k = 0.035$$

Example 4

The amount of a radioactive substance is given by $A = A_0 e^{-kt}$.

In 3 minutes 10 grams of Bismuth reduce to 9 grams through radioactive decay.

(a) Find k

(b) Find the half-life of bismuth.

Solution to (b):

$$A = A_0 e^{-kt}$$

$A_0 = 2$

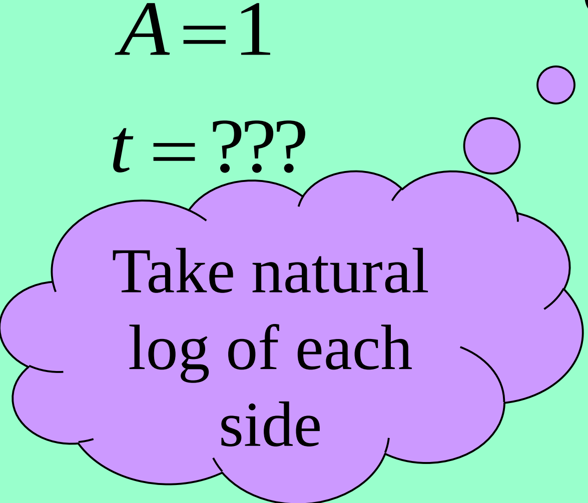
$A = 1$

$t = ???$

$$1 = 2\exp(-0.035t)$$

$$\exp(-0.035t) = \tfrac{1}{2} = 0.5$$

$$-0.035t = \ln(0 \cdot 5)$$



Example 4

The amount of a radioactive substance is given by $A = A_0 e^{-kt}$.

In 3 minutes 10 grams of Bismuth reduce to 9 grams through radioactive decay.

(a) Find k

(b) Find the half-life of bismuth.

Solution to (b):

$$A_0 = 2$$

$$A = 1$$

$$t = ???$$

$$A = A_0 e^{-kt}$$

$$t = \frac{\ln(0.5)}{-0 \cdot 035}$$

$$t = 19 \cdot 8 \text{ min}$$

Heinemann, p.290, EX 15H, Q7