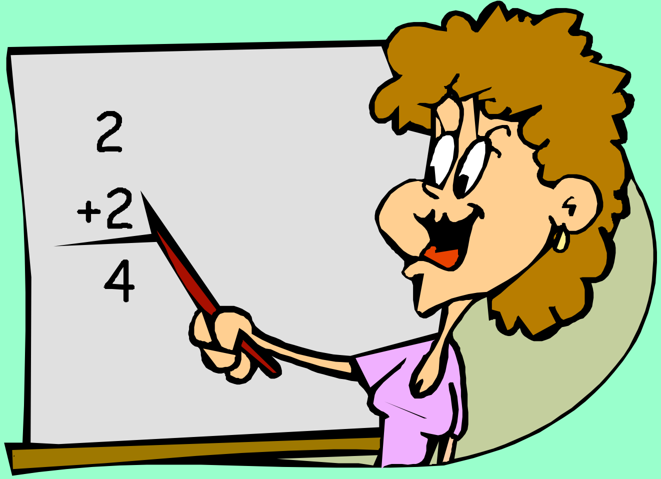

5.

Integrating trig functions with brackets

$$\int \sin(ax+b)\ dx = -\frac{1}{a}\cos(ax+b)+C$$

$$\int \cos(ax+b)\ dx = \frac{1}{a}\sin(ax+b)+C$$

Trig functions with brackets

We have seen that if $f(x)=\sin(2x+1)$ then $f'(x)=2\cos(2x+1)$

If I want to integrate $\cos(2x+1)$ my answer must be $\dfrac{\sin(2x+1)}{2}$

This leads to the following rules:

$$\int \sin(ax+b)\ dx = -\frac{1}{a}\cos(ax+b)+C = -\frac{\cos(ax+b)}{a}+C$$

$$\int \cos(ax+b)\ dx = \frac{1}{a}\sin(ax+b)+C = \frac{\sin(ax+b)}{a}+C$$

Example 1

Find:

NAB

$(a)\ \int 3\cos x\ dx$ $(b)\ \int \sin(4x+5)\ dx$ $(c)\ \int (x^3+\cos(8-x))\ dx$

Solution:

Only int functions of x

$(a)\ \int 3\cos x\ dx$ $(b)\ \int \sin(4x+5)\ dx$ $(c)\ \int (x^3+\cos(8-x))\ dx$

$= 3\times\int \cos x\ dx$ $= -\dfrac{\cos(4x+5)}{4}$ $= \int x^3 + \int \cos(8-x)\ dx$

SICk

$= 3\times\dfrac{\sin x}{1}$ $= -\tfrac{1}{4}\cos(4x+5)+C$ $= \dfrac{x^4}{4} + \left(\dfrac{\sin(8-x)}{-1}\right) + C$

$= 3\sin x + C$

$= \tfrac{1}{4}x^4 - \sin(8-x) + C$

Heinemann,
p.275, EX 14K, Q1(a) to (h)

This is not the end

Example 2

The curve $y = f(x)$ passes through the point $\left(\frac{\pi}{12},\ 1\right)$ and $f'(x) = \cos 2x$.
Find $f(x)$. **(3)**

Solution:

If $f'(x) = \cos 2x$ then $f(x) = \int \cos 2x \, dx$

$$= \frac{\sin 2x}{2} + C$$

For the point $\left(\frac{\pi}{12}, 1\right)$

$$1 = \frac{\sin\left(\frac{2\pi}{12}\right)}{2} + C$$

$$2 = \sin\left(30^0\right) + C$$

$$C = 2 - \tfrac{1}{2} = \tfrac{3}{2}$$

$$\Rightarrow f(x) = \frac{\sin 2x}{2} + \frac{3}{2} = \frac{\sin 2x + 3}{2}$$