

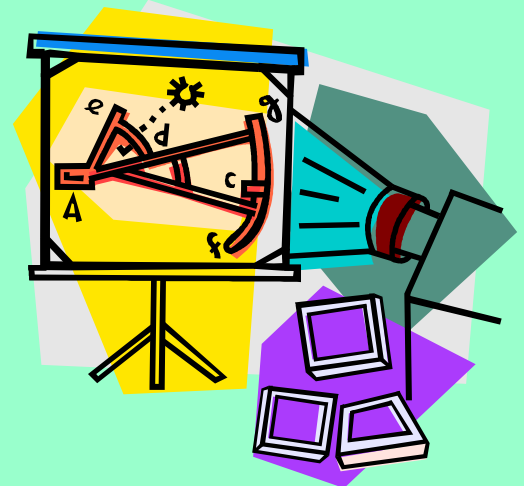


5.

Finding m and c



$$u_{n+1} = mu_n + c$$



What if we are asked to find the recurrence relation?

In all the previous examples we have been **given** a recurrence relation and asked to determine values from the given RR.

What if we are given values generated by the RR and asked to find the recurrence relation which generated them?

Given the general form $u_{n+1} = mu_n + c$

we need to work out m and c .

To do so we need at least 3 values generated by the RR. Two values would give us only one equation with 2 unknowns (m and c).

Three values allows us to form 2 equations both containing the same unknowns (m and c) i.e **simultaneous equations**.

Finding the recurrence relation

Copy the following:

Example 1

A recurrence relation is defined by $u_{n+1} = mu_n + c$

Find the values of m and c if : $u_5 = 4$ $u_6 = -2$ and $u_7 = 16$

Solution :

1. Write down the RR for the first pair of values.

$$u_6 = mu_5 + c$$

$$-2 = m(4) + c$$

$$4m + c = -2$$

2. Write down the RR for the other pair of values.

$$u_7 = mu_6 + c$$

$$16 = m(-2) + c$$

$$-2m + c = 16$$

Finding the recurrence relation

Copy the following:

Example 1

A recurrence relation is defined by $u_{n+1} = mu_n + c$

Find the values of m and c if : $u_5 = 4$ $u_6 = -2$ and $u_7 = 16$

Solution :

3. Create and solve simultaneous equations.

$$\left. \begin{array}{l} 4m + c = -2 \\ -2m + c = 16 \end{array} \right\} \begin{array}{l} 1. \\ 2. \end{array}$$

$$6m = -18$$

$$m = -3$$

Sub $m = -3$ into eqn 1.

$$4(-3) + c = -2$$

$$-12 + c = -2$$

$$c = 10$$

Finding the recurrence relation

Copy the following:

Example 1

A recurrence relation is defined by $u_{n+1} = mu_n + c$

Find the values of m and c if : $u_5 = 4$ $u_6 = -2$ and $u_7 = 16$

Solution :

4. Make statement and answer question

So $m = -3$, $c = 10$ and

$$u_{n+1} = -3u_n + 10$$

Example 2

The outstanding debt on a credit card after the first three consecutive months after the borrowing began were £578, £555.34 and £532.

Each month interest is charged on the outstanding balance and a fixed sum has to be paid at the end of each month.

Find the interest rate and the fixed monthly payment and find the original amount outstanding.

Solution :

1. Write down information contained In the question.
2. Write down the RR for the first pair of values.

$$u_1 = 578 \quad u_2 = 555.34$$
$$u_3 = 532$$

$$u_2 = mu_1 + c$$

$$555.34 = m(578) + c$$

$$578m + c = 555.34$$

Example 2

The outstanding debt on a credit card after the first three consecutive months after the borrowing began were £578, £555.34 and £532.

Each month interest is charged on the outstanding balance and a fixed sum has to be paid at the end of each month.

Find the interest rate and the fixed monthly payment and the original amount outstanding.

Solution :

3. Write down the RR for the other pair of values.

$$u_1 = 578 \quad u_2 = 555.34$$

$$u_3 = 532$$

$$u_3 = mu_2 + c$$

$$532 = m(555.34) + c$$

$$555.34m + c = 532$$

Example 2

The outstanding debt on a credit card after the first three consecutive months after the borrowing began were £578, £555.34 and £532.

Each month interest is charged on the outstanding balance and a fixed sum has to be paid at the end of each month.

Find the interest rate and the fixed monthly payment and the original amount outstanding.

Solution :

4. Create and solve simultaneous equations.

$$\left. \begin{array}{l} 578.00m + c = 555.34 \\ 555.34m + c = 532 \end{array} \right\} \begin{array}{l} 1. \\ 2. \end{array}$$

$$22.66m = 23.34$$

$$m = 1.03$$

Sub $m = 1.03$ into eqn 1.

$$578(1.03) + c = 555.34$$

$$595.34 + c = 555.34$$

$$C = -40$$

Example 2

The outstanding debt on a credit card after the first three consecutive months after the borrowing began were £578, £555.34 and £532.

Each month interest is charged on the outstanding balance and a fixed sum has to be paid at the end of each month.

Find the interest rate and the fixed monthly payment and the original amount outstanding.

Solution :

5. Make statement and answer question

$$m = 1.03$$

$$C = -40$$

6. Use this RR to find starting value

So monthly interest rate is 3% and fixed monthly payment is £40 and

$$u_{n+1} = 1.03u_n - 40$$

$$u_1 = 1.03u_0 - 40$$

$$578 = 1.03u_0 - 40$$

$$618 = 1.03u_0$$

$$u_0 = \frac{618}{1.03} = 600$$

Example 2

The outstanding debt on a credit card after the first three consecutive months after the borrowing began were £578, £555.34 and £532.

Each month interest is charged on the outstanding balance and a fixed sum has to be paid at the end of each month.

Find the interest rate and the fixed monthly payment and the original amount outstanding.

Solution :

7. Make statement and answer question

$$u_0 = 600$$

So original debt was £600

Booklet , p.11, EX 7