5.

Area Above and Below the x-axis

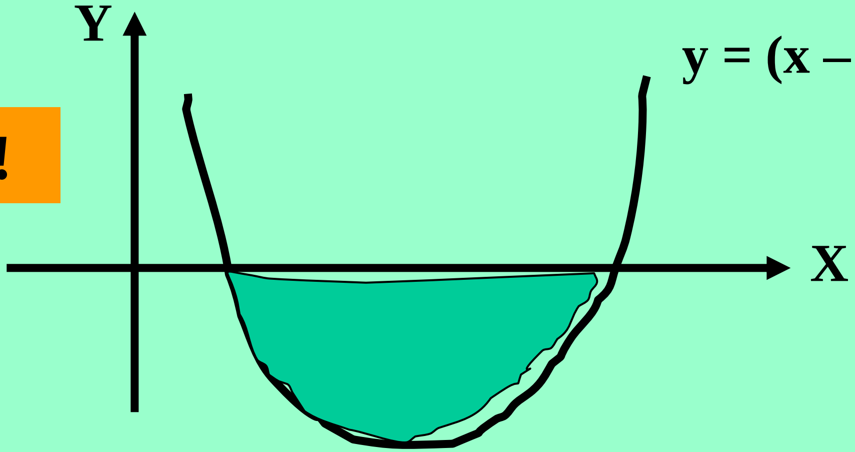
INTEGRATION & AREA

So far we have looked at integrating functions to find the area under under a given curve between two points where f(x) is always positive (or zero).

Now we will consider what happens when the area being evaluated is, at least in part, in the negative region of the graph.

Example 1

NB: need limits!



1. Find limits by setting f(x) = 0 and factorising

Curve cuts X-axis when (x – 1)(x – 6) = 0 so x = 1 or x = 6

2. Prepare and then evaluate integral between these limits

$$Area = \int_1^6 (x-1)(x-6)\,dx$$

$$= \int_1^6 (x^2 - 7x + 6)\,dx$$

$$= \left[\tfrac{1}{3}x^3 - \tfrac{7}{2}x^2 + 6x \right]_1^6$$

$$= (72 - 126 + 36) - \left(\tfrac{1}{3} - \tfrac{7}{2} + 6\right)$$

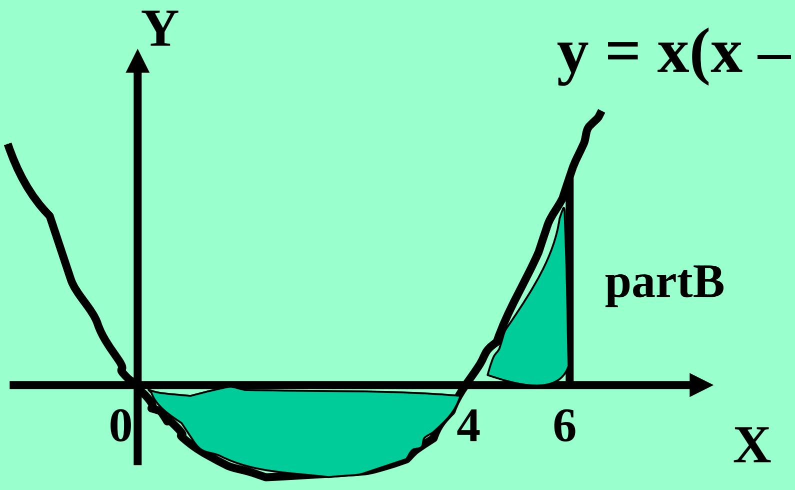
$$= -20\tfrac{5}{6} \text{ units}^2 \quad (**)$$

(**) Area can't be negative.

Negative sign indicates area is below X-axis.

Actual area = $20\tfrac{5}{6}$ units^2

Example 2



$$Area = \int_0^6 (x(x-4))dx$$

$$Area = \int_0^6 (x^2 - 4x)dx$$

$$= \left[\tfrac{1}{3}x^3 - 2x^2 \right]_0^6$$

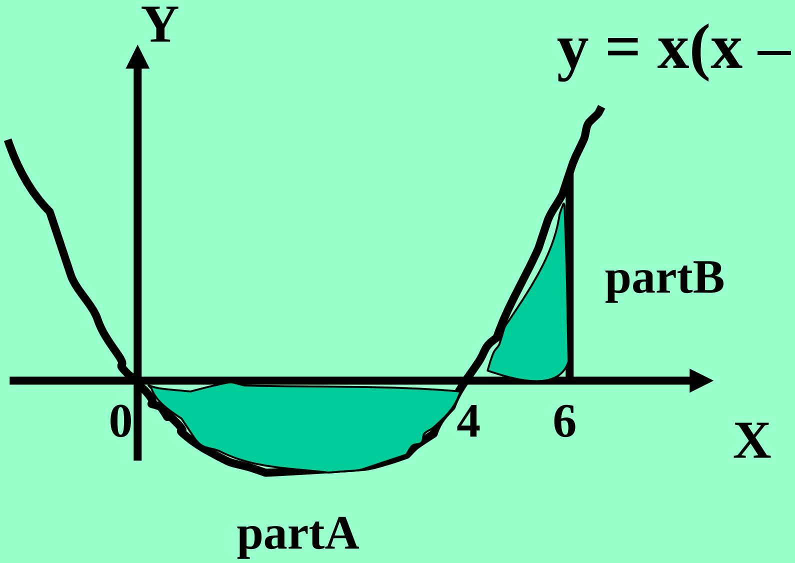
$$= (72 - 72) - 0$$

$$= \underline{0}$$

It is obvious the total area is not zero but the equal magnitude positive and negative parts have cancelled each other out.

Hence the need to do each bit separately.

Example 2



Need to find each section separately !

$$AreaA = \int_0^4 (x(x-4))dx$$

$$Area = \int_0^4 (x^2 - 4x)dx$$

$$= \left[\tfrac{1}{3}x^3 - 2x^2 \right]_0^4$$

$$= (21\tfrac{1}{3} - 32) - 0$$

$$= -10\tfrac{2}{3} \quad (\text{really } 10\tfrac{2}{3})$$

$$AreaB = \int_4^6 (x^2 - 4x)dx$$

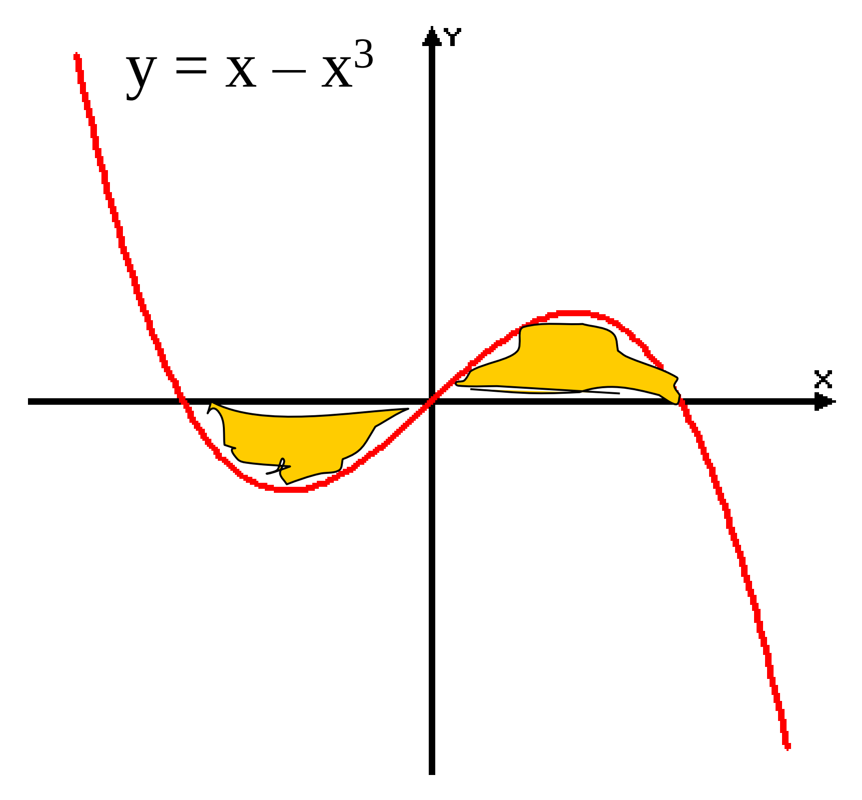
$$= \left[\tfrac{1}{3}x^3 - 2x^2 \right]_4^6$$

$$= (72 - 72) - (21\tfrac{1}{3} - 32)$$

$$= 10\tfrac{2}{3}$$

Total = $10\tfrac{2}{3} + 10\tfrac{2}{3} = 21\tfrac{1}{3}$ units^2

Example 3



NB: need limits!

Polynomial may need synthetic division to find all limits

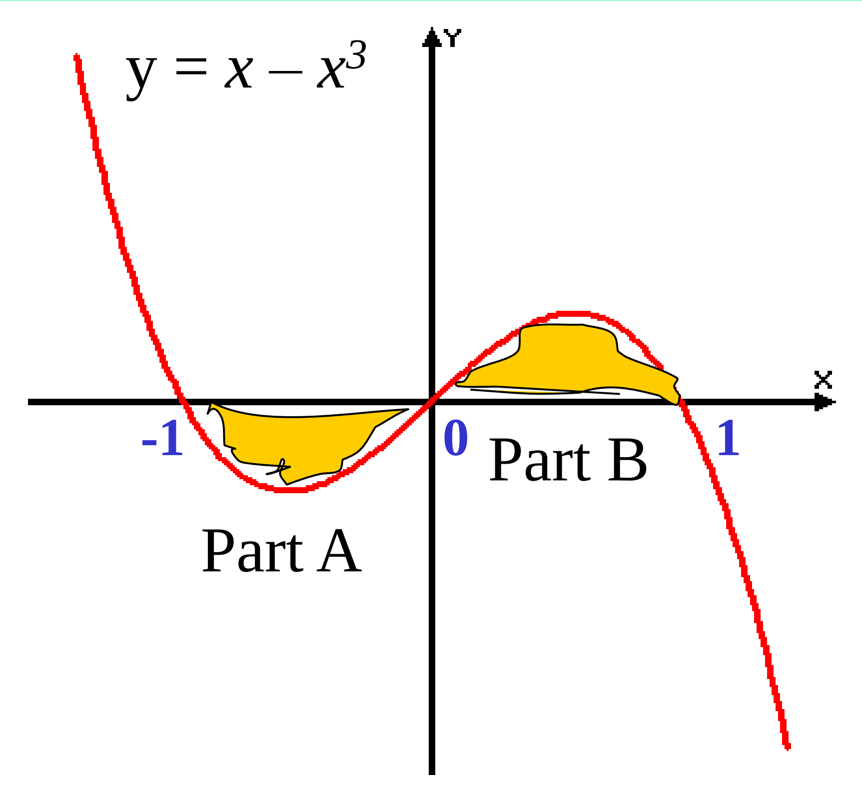
1. Find limits by setting f(x) = 0 and factorising

$$x - x^3 = 0 \rightarrow x(1 - x^2) = 0$$

So $x = 0$ or $x^2 = 1$

$$x = \pm 1$$

Example 3



NB: need limits!

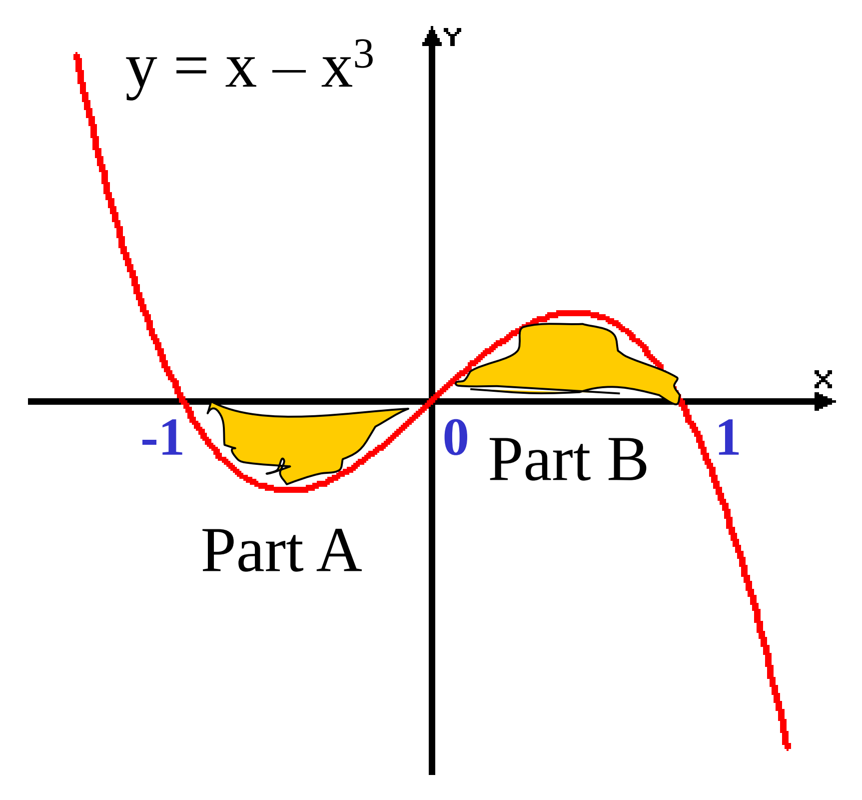
So $x = 0$ $\qquad x = \pm 1$

2. Prepare and then evaluate integral between these limits

$$AreaA = \int_{-1}^{0} (x - x^3)dx \quad = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{-1}^{0} \quad = (0) - \left(\frac{1}{2} - \frac{1}{4}\right) \quad = 0 - \left(\frac{2}{4} - \frac{1}{4}\right)$$

= -¼ units²

Example 3



NB: need limits!

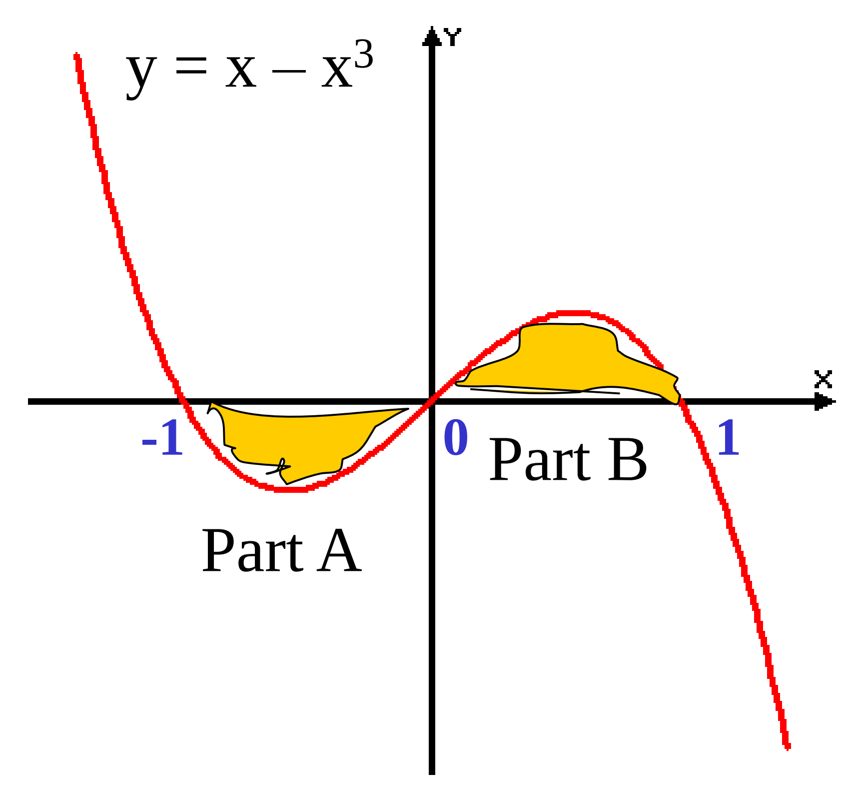
So $x = 0$ $\qquad x = \pm 1$

2. Prepare and then evaluate integral between these limits

$$AreaB = \int_0^1 (x - x^3)dx \quad = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 \quad = \left(\frac{1}{2} - \frac{1}{4}\right) - (0) \quad = \left(\frac{2}{4} - \frac{1}{4}\right) - 0$$

= ¼ units²

Example 3



NB: need limits!

So $x = 0$ $\qquad x = \pm 1$

So total area = **¼ units²** **+ ¼ units²** **= ½ units²**

Heinemann, p.177, EX 9N, Q1 & 2