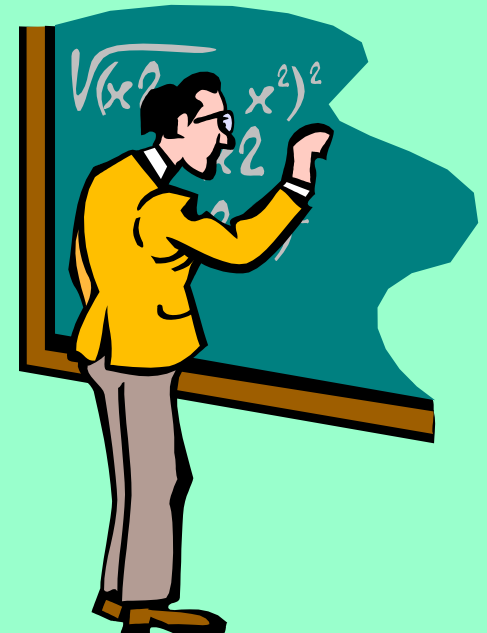
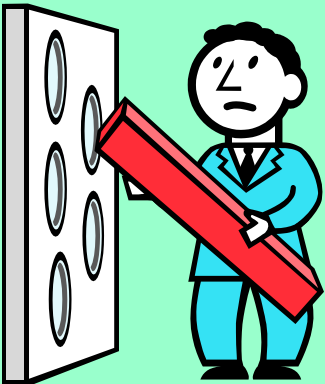




4.

# Sketching a quadratic

$$y = k(x - a)^2 + b$$



## When is the completed square form useful?

Completed square form is useful because it tells immediately what the turning point of a quadratic is. This is very useful when the parabola does not cut the x-axis.

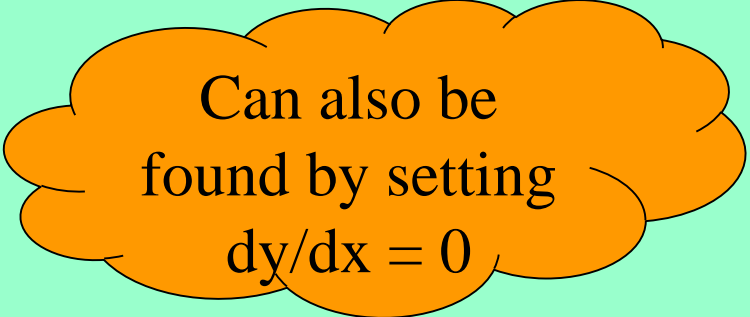
We can also use the completed square form to help sketch a parabola and that is what we are going to look at now.

When sketching a parabola there are 3 things we need to know:

1. Turning point and its nature

2. Y-intercept ( $X = 0$ )

3. Roots ( $Y = 0$ )



Can also be  
found by setting  
 $dy/dx = 0$

## Example 1

Sketch the graph of  $y = x^2 - 4x - 12$

### Solution:

1. Write the equation in completed square form.
2. Establish turning point.
3. Find y-intercept ( $X = 0$ )

$$y = x^2 - 4x - 12$$

$$= [(x - 2)^2 - 4] - 12$$

$$y = (x - 2)^2 - 16$$

So minimum tp occurs at:

$$(2, -16)$$

$$y(0) = (0)^2 - 4(0) - 12 = -12$$

Y-Intercept is (0, -12)

## Example 1

Sketch the graph of  $y = x^2 - 4x - 12$

**Solution:**

4. Find Roots ( $Y = 0$ )

5. Now use this information to plot points and join to make a parabola.

$$y = x^2 - 4x - 12$$

$$y = (x - 2)^2 - 16$$

$$\Rightarrow (x - 2)^2 - 16 = 0$$

$$\Rightarrow (x - 2)^2 = 16$$

$$\Rightarrow (x - 2) = \pm 4$$

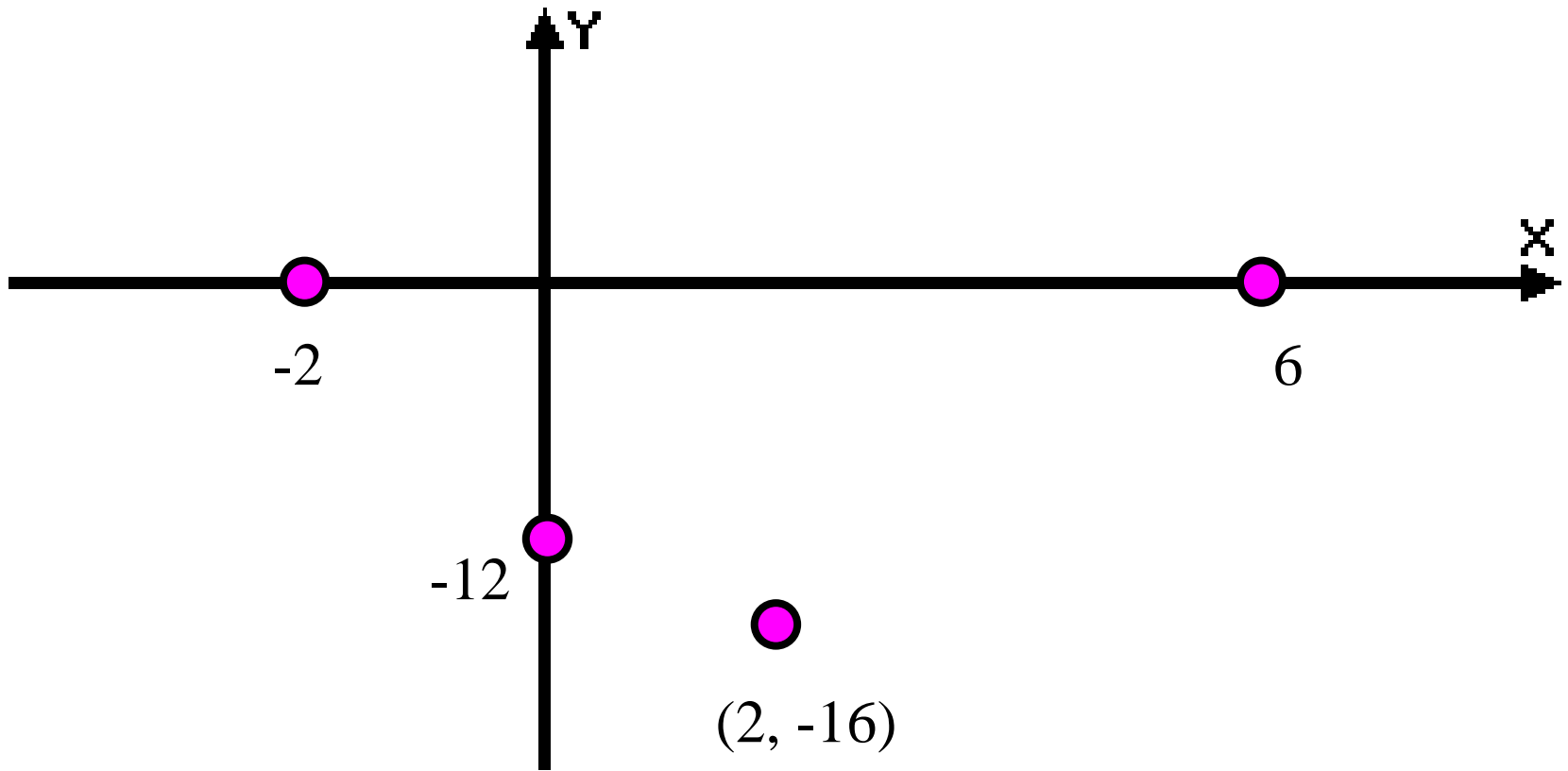
$$\Rightarrow x = 4 + 2 \quad \text{or} \quad x = -4 + 2$$

$$\Rightarrow x = 6 \quad \text{or} \quad x = -2$$

So Roots are:

$$(6, 0)$$

$$(-2, 0)$$



So minimum tp occurs at:

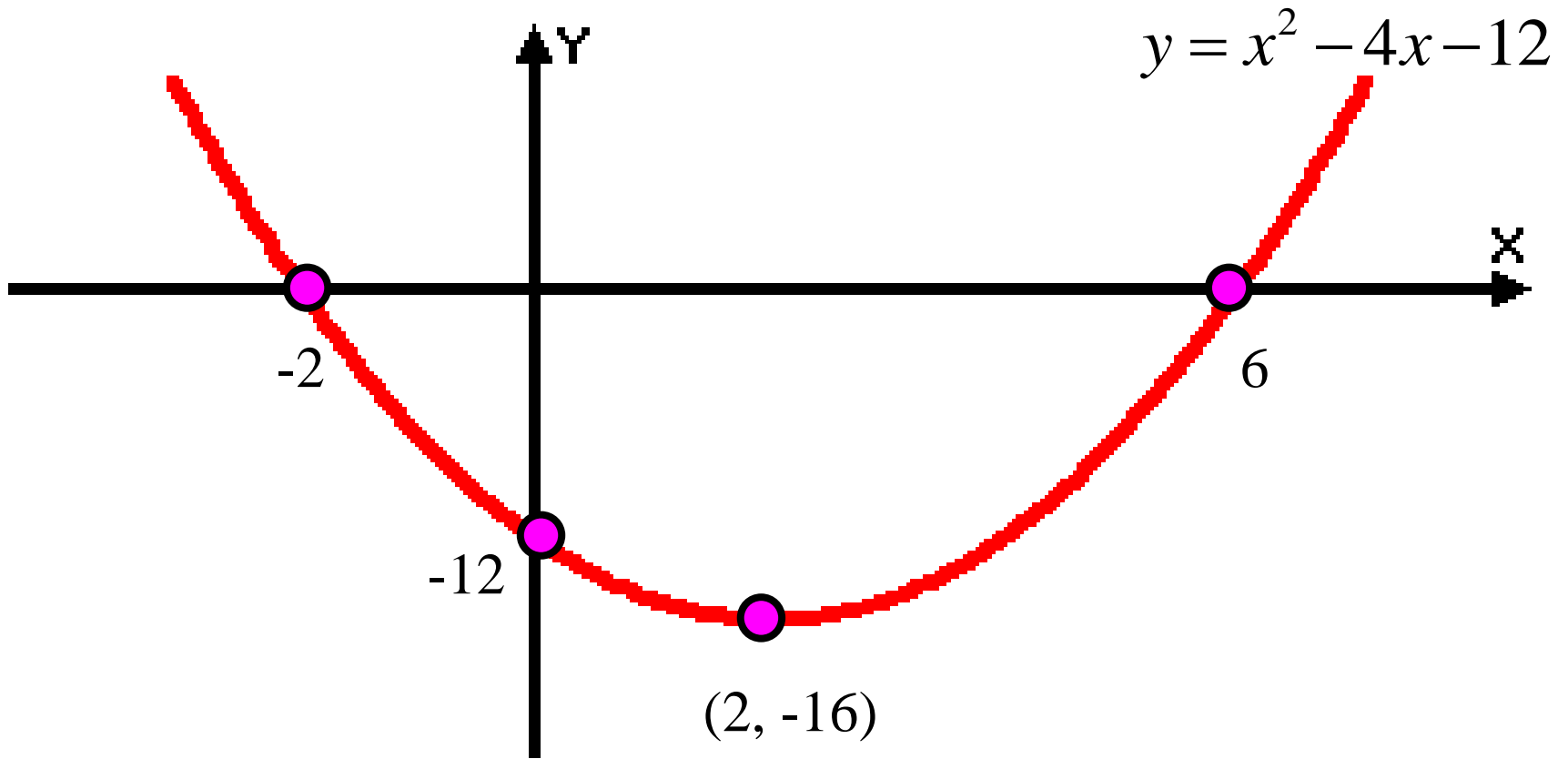
$(2, -16)$

Y-Intercept is  $(0, -12)$

So Roots are:

$(6, 0)$

$(-2, 0)$



So minimum tp occurs at:

$(2, -16)$

Y-Intercept is  $(0, -12)$

So Roots are:

$(6, 0)$

$(-2, 0)$

Heinemann, p.146, EX 8D,  
Q2, (a) & (f)

## Example 2 (No Roots)

Sketch the graph of  $y = x^2 - 6x + 22$

### Solution:

1. Write the equation in completed square form.
2. Establish turning point.
3. Find y-intercept ( $X = 0$ )

$$y = x^2 - 6x + 22$$

$$= [(x - 3)^2 - 9] + 22$$

$$y = (x - 3)^2 + 13$$

So minimum tp occurs at:

$$(3, 13)$$

$$y(0) = (0)^2 - 6(0) + 22 = 22$$

Y-Intercept is (0, 22)



## Example 2

Sketch the graph of  $y = x^2 - 6x + 22$

**Solution:**

4. Find Roots ( $Y = 0$ )

Can't square  
root negative!  
So no roots

5. Now use this information  
to plot points and join to  
make a parabola.

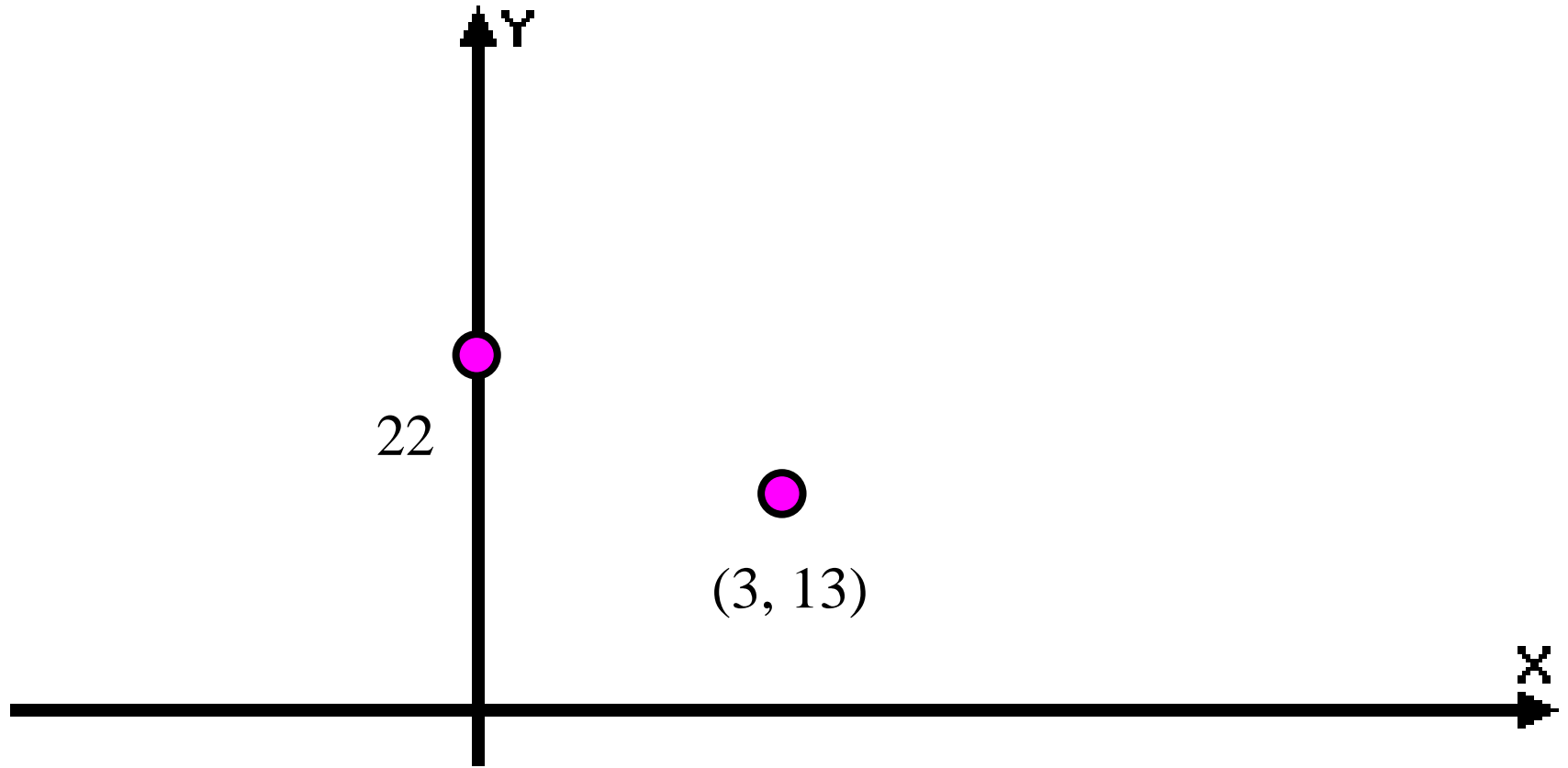
$$y = x^2 - 6x + 22$$

$$y = (x - 3)^2 + 13$$

$$\Rightarrow (x - 3)^2 + 13 = 0$$

$$\Rightarrow (x - 3)^2 = -13$$

So no Roots



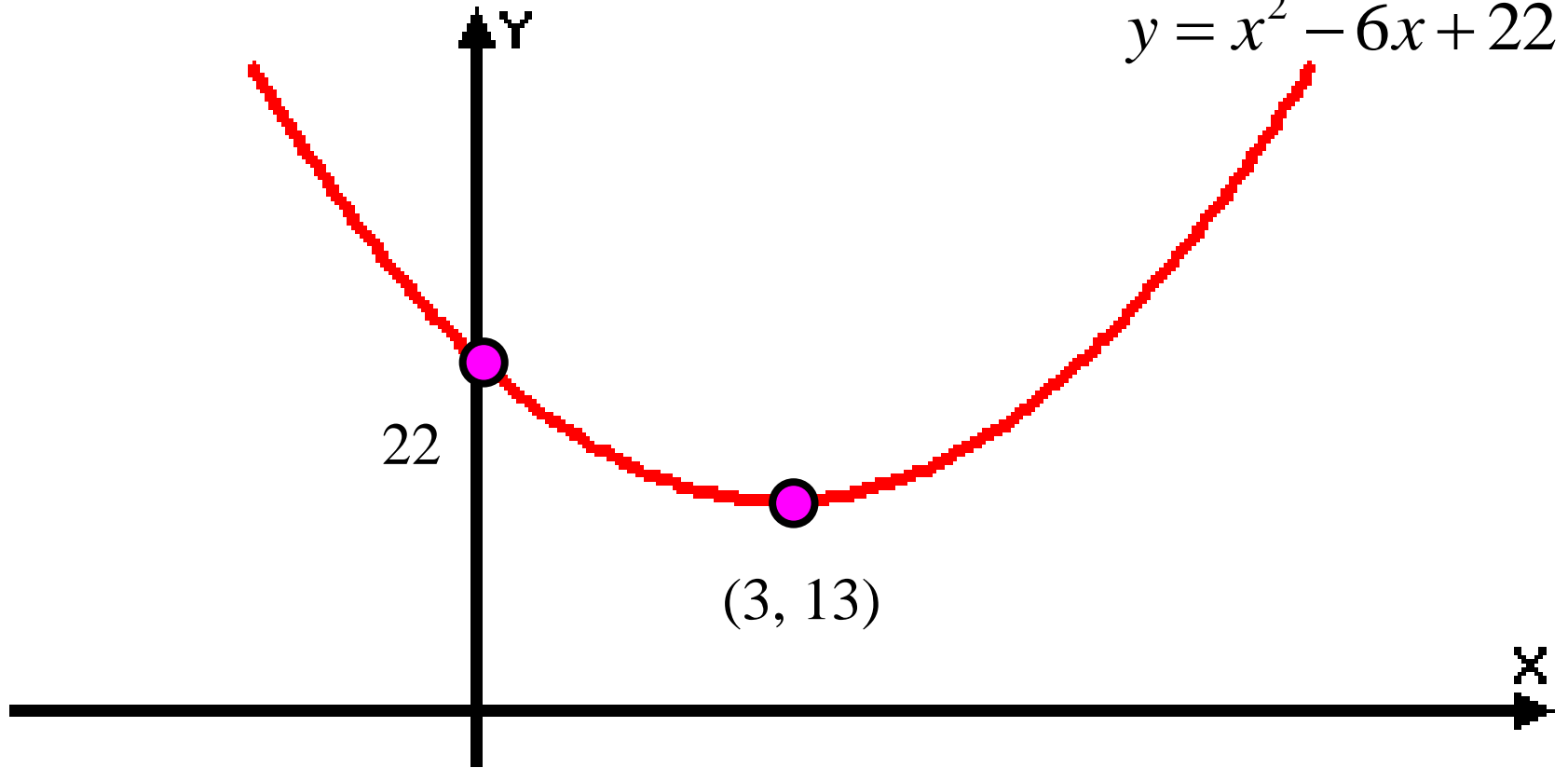
So minimum tp occurs at:

$(3, 13)$

Y-Intercept is  $(0, 22)$

So no roots

$$y = x^2 - 6x + 22$$



So minimum tp occurs at:

$(3, 13)$

Y-Intercept is  $(0, 22)$

So no roots

Heinemann, p.146, EX 8D,  
Q2 (c) & (e)

### Example 3 (Negatives)

Sketch the graph of  $y = 24 - 2x - x^2$

#### Solution:

1. Write the equation in completed square form.
2. Establish turning point.
3. Find y-intercept ( $X = 0$ )

$$y = -1(x^2 + 2x - 24)$$
$$= -1([(x+1)^2 - 1] - 24)$$

$$y = -1[(x+1)^2 - 25]$$

$$y = -(x+1)^2 + 25$$

So maximum tp occurs at:

$$(-1, 25)$$

$$y(0) = 24 - 2(0) - (0)^2 = 24$$

Y-Intercept is (0, 24)

### Example 3 (Negatives)

Sketch the graph of  $y = 24 - 2x - x^2$

**Solution:**

4. Find Roots ( $Y = 0$ )

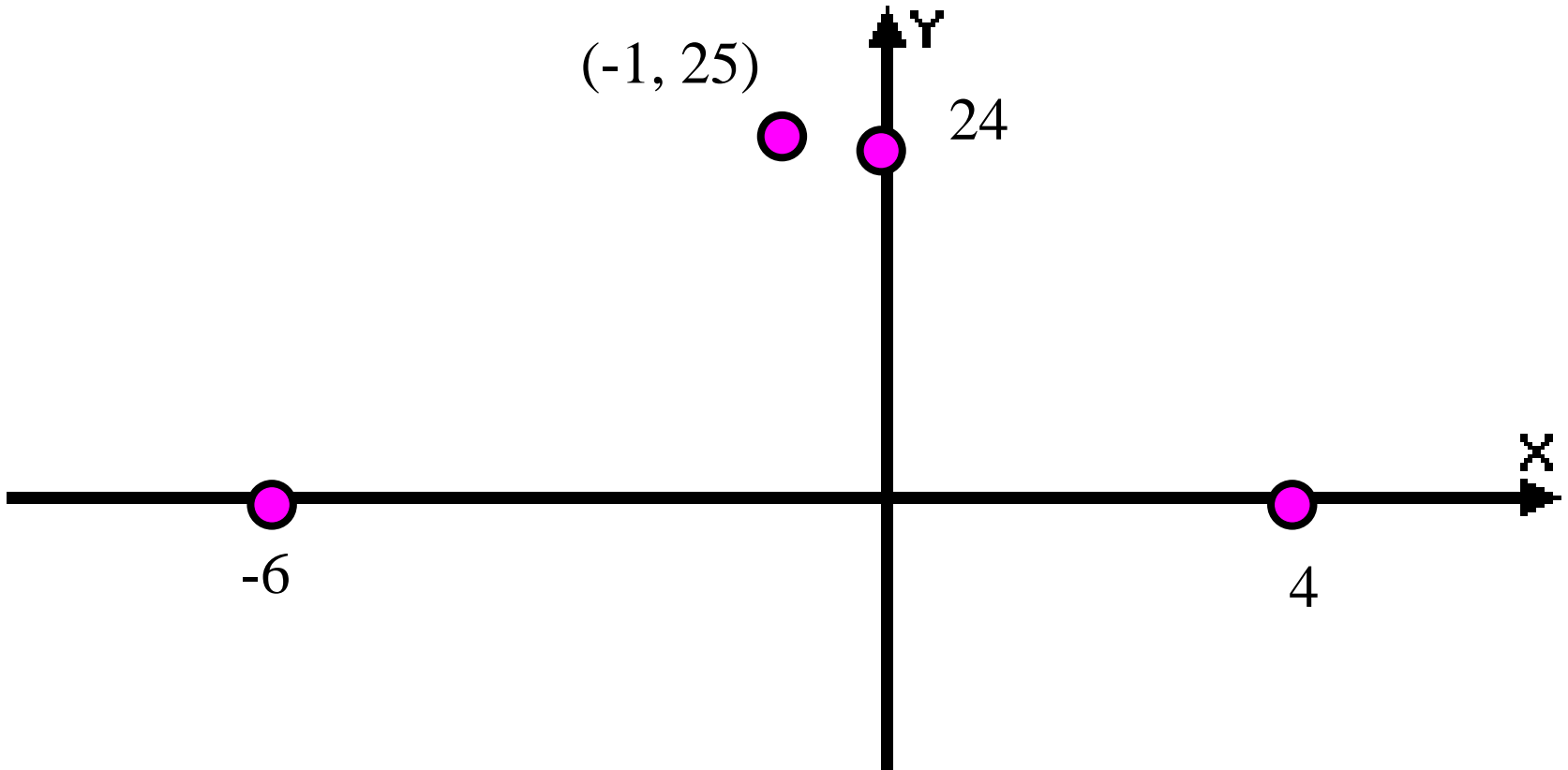
5. Now use this information to plot points and join to make a parabola.

$$\begin{aligned}y &= 24 - 2x - x^2 \\y &= -(x+1)^2 + 25 \\ \Rightarrow -(x+1)^2 + 25 &= 0 \\ \Rightarrow \ominus(x+1)^2 &= -25 \\ \Rightarrow (x+1)^2 &= 25 \\ \Rightarrow (x+1) &= \pm 5 \\ \Rightarrow x = 5 - 1 \quad \text{or} \quad x = -5 - 1 \\ \Rightarrow x = 4 \quad \quad \quad \text{or} \quad x = -6\end{aligned}$$

So Roots are:

(4, 0)

(-6, 0)



So maximum tp occurs at:

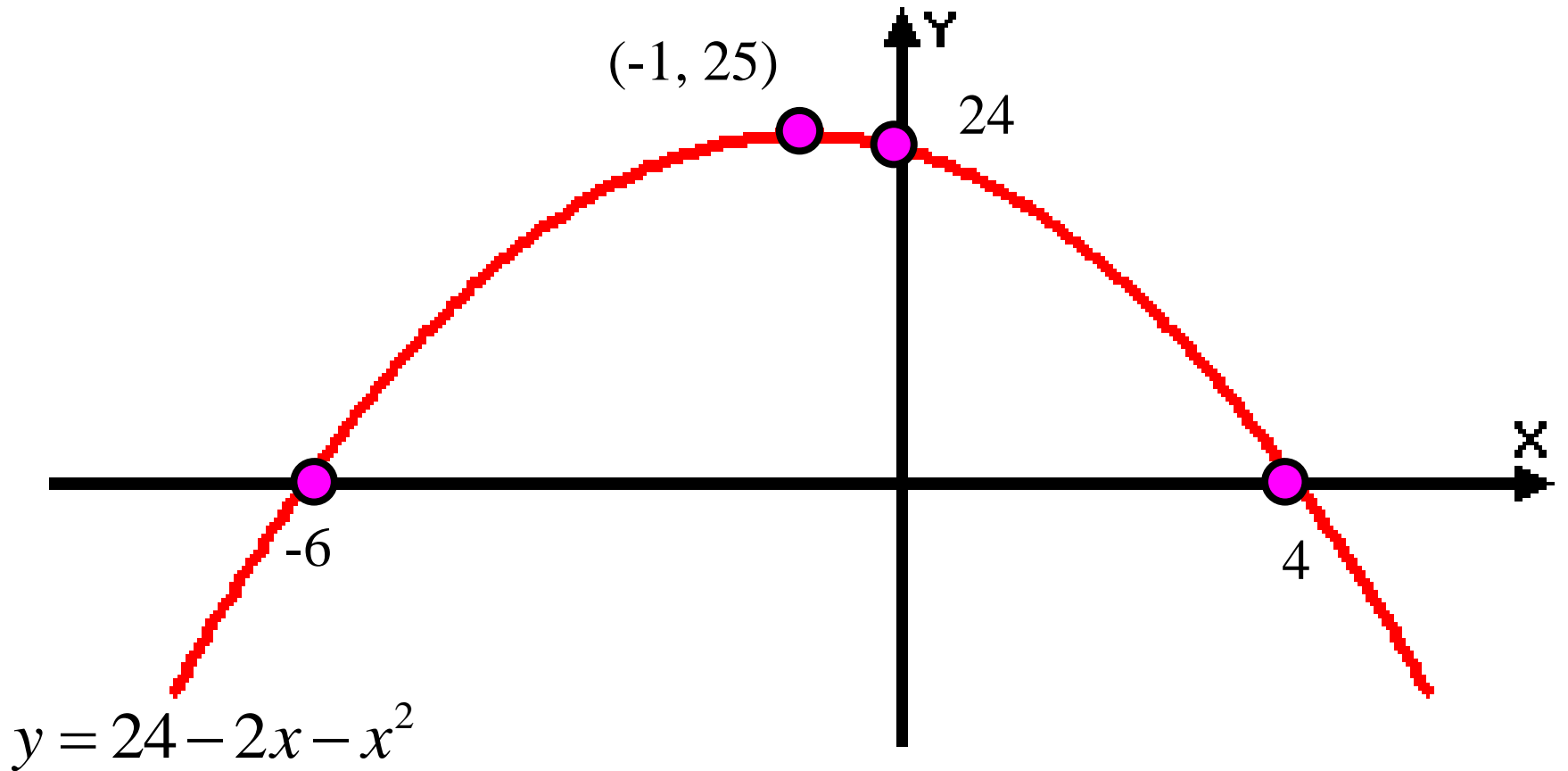
$(-1, 25)$

Y-Intercept is  $(0, 24)$

So Roots are:

$(4, 0)$

$(-6, 0)$



So maximum tp occurs at:

$(-1, 25)$

Y-Intercept is  $(0, 24)$

So Roots are:

$(4, 0)$

$(-6, 0)$



Heinemann, p.146, EX 8D,  
Q2, (b) & (d)