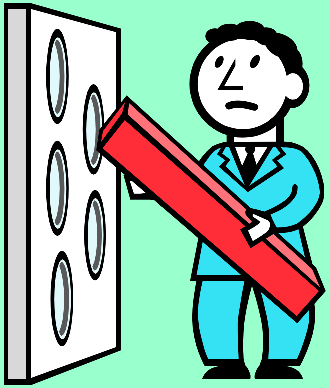
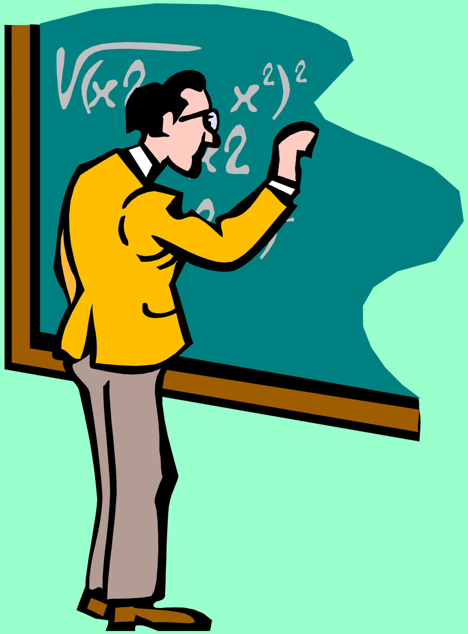
4. Sketching a quadratic

$$y = k(x - a)^2 + b$$





When is the completed square form useful?

Completed square form is useful because it tells immediately what the turning point of a quadratic is. This is very useful when the parabola does not cut the x-axis.

We can also use the completed square form to help sketch a parabola and that is what we are going to look at now.

When sketching a parabola there are 3 things we need to know:

1. Turning point and its nature

2. Y-intercept (X = 0)

3. Roots (Y = 0)

Can also be found by setting dy/dx = 0

Example 1

$$y = a(x+p)^2 + q = ax^2 + 2apx + ap^2 + q$$

Sketch the graph of $y = x^2 - 4x - 12$

Solution:

1. Write the equation in completed square form.

2. Establish turning point.

3. Find y-intercept (X = 0)

$$y = x^2 - 4x - 12$$

a = 1

2ap = -4

p = -2

ap^2 + q = -12

4 + q = -12

q = -16

$$y = (x-2)^2 - 16$$

So minimum tp occurs at:

(2 , -16)

$$y(0) = (0)^2 - 4(0) - 12 = -12$$

Y-Intercept is (0, -12)

Example 1

Sketch the graph of $y = x^2 - 4x - 12$

Solution:

4. Find Roots (Y = 0)

$$y = x^2 - 4x - 12$$

$$y = (x-2)^2 - 16$$

$$\Rightarrow (x-2)^2 - 16 = 0$$

$$\Rightarrow (x-2)^2 = 16$$

$$\Rightarrow (x-2) = \pm 4$$

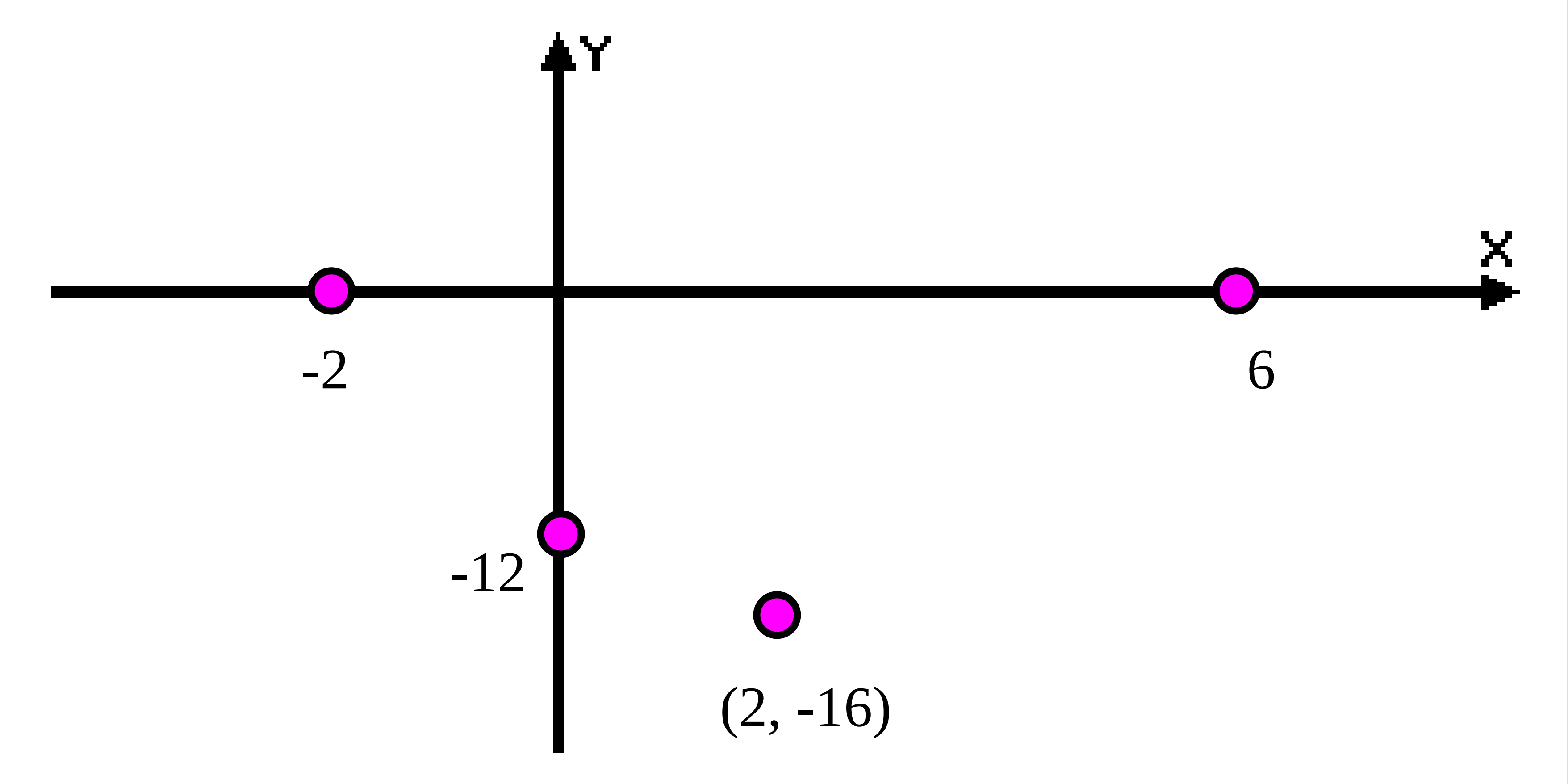
$$\Rightarrow x = 4 + 2 \quad or \quad x = -4 + 2$$

$$\Rightarrow x = 6 \quad or \quad x = -2$$

So Roots are:

(6 , 0) (-2 , 0)

5. Now use this information to plot points and join to make a parabola.



So minimum tp occurs at:

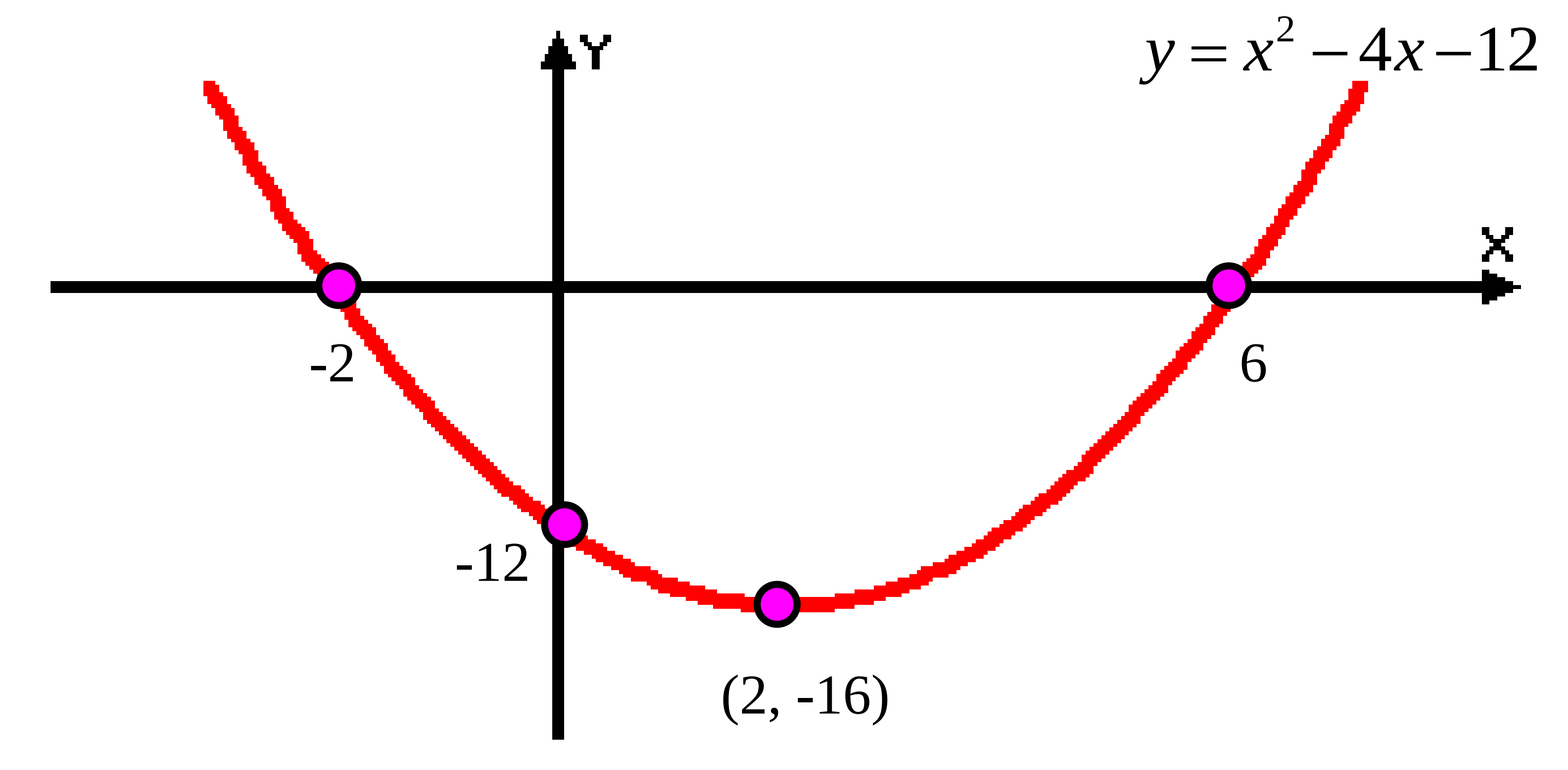
(2 , -16)

Y-Intercept is (0, -12)

So Roots are:

(6 , 0)

(-2 , 0)



So minimum tp occurs at:

(2 , -16)

Y-Intercept is (0, -12)

So Roots are:

(6 , 0)

(-2 , 0)

Heinemann, p.146, EX 8D,
Q2, (a) & (f)

Example 2

(No Roots)

$$y = a(x+p)^2 + q \quad = ax^2 + 2apx + ap^2 + q$$

Sketch the graph of $y = x^2 - 6x + 22$

Solution:

1. Write the equation in completed square form.

2. Establish turning point.

3. Find y-intercept (X = 0)

$$y = x^2 - 6x + 22$$

a = 1 | **2ap = -6** | **ap^2 + q = 22**

p = -3

9 + q = 22

q = 13

$$y = (x-3)^2 + 13$$

So minimum tp occurs at:

(3 , 13)

$$y(0) = (0)^2 - 6(0) + 22 = 22$$

Y-Intercept is (0, 22)

Example 2

Sketch the graph of $y = x^2 - 6x + 22$

Solution:

4. Find Roots (Y = 0)

Can't square root negative! So no roots

$$y = x^2 - 6 + 22$$

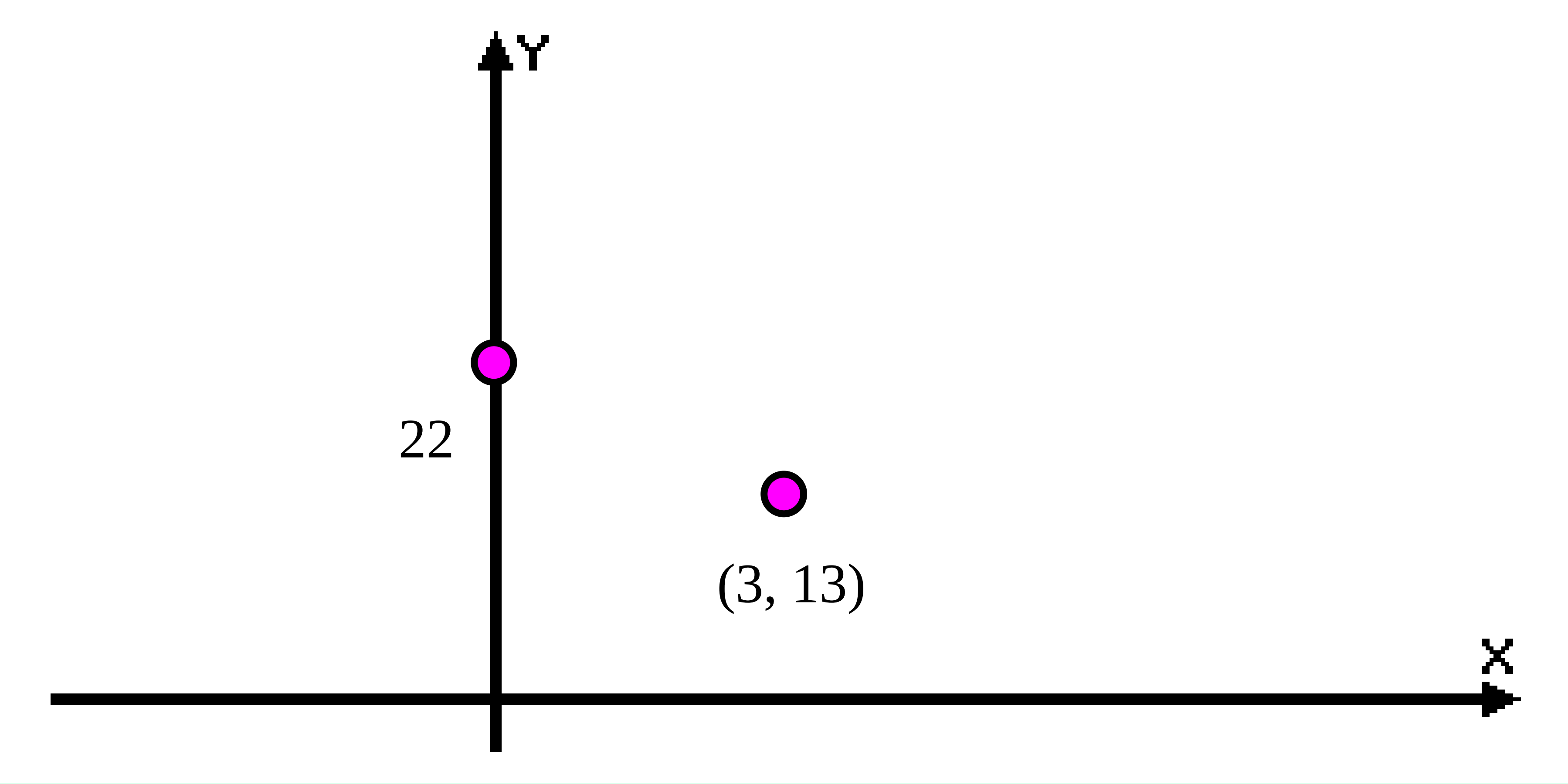
$$y = (x-3)^2 + 13$$

$$\Rightarrow (x-3)^2 + 13 = 0$$

$$\Rightarrow (x-3)^2 = -13$$

So no Roots

5. Now use this information to plot points and join to make a parabola.

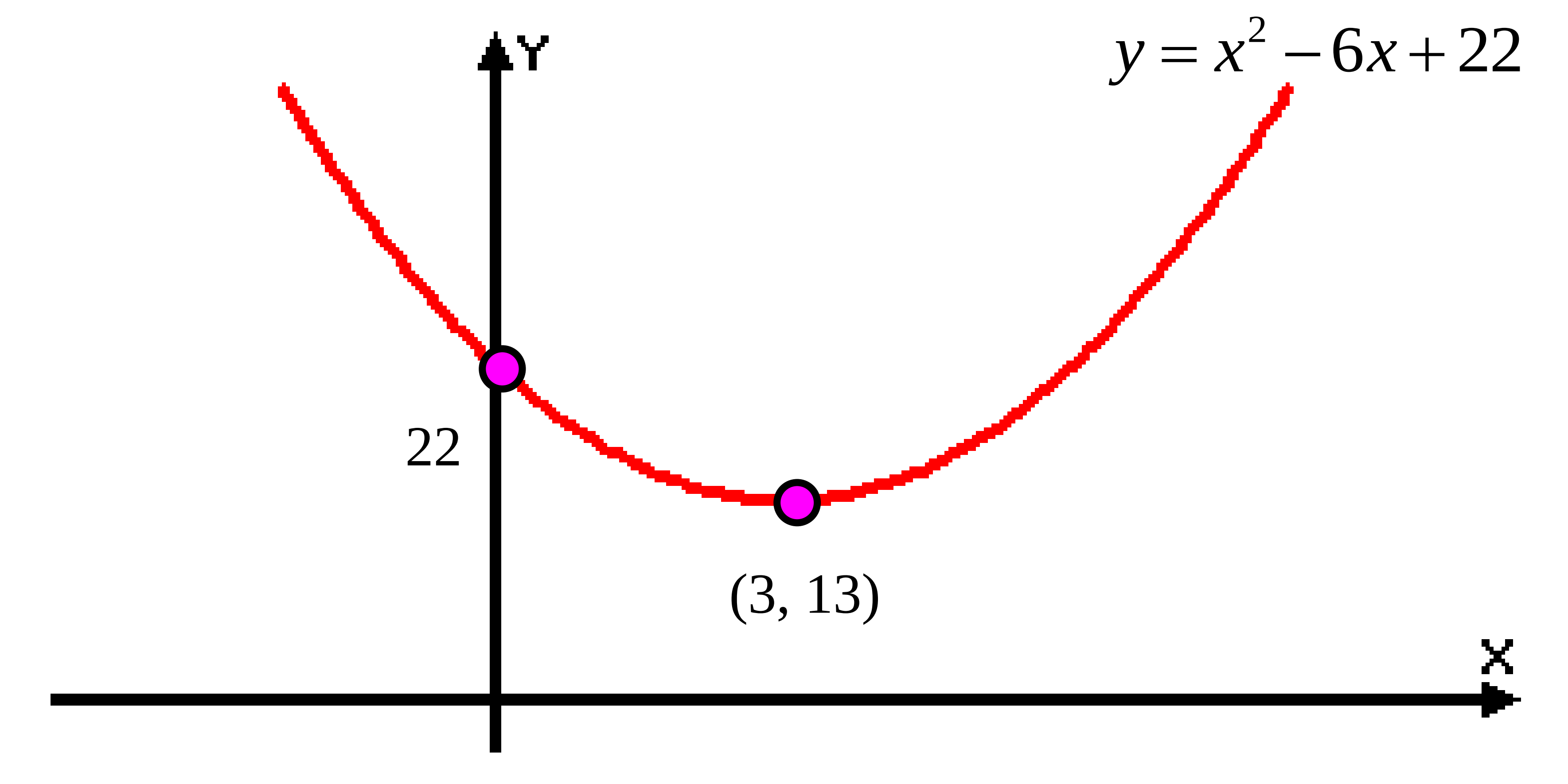


So minimum tp occurs at:

(3 , 13)

Y-Intercept is (0, 22)

So no roots



So minimum tp occurs at:

(3 , 13)

Y-Intercept is (0, 22)

So no roots

Heinemann, p.146, EX 8D,
Q2 (c) & (e)

Example 3

(Negatives)

$$y = a(x+p)^2 + q \quad = ax^2 + 2apx + ap^2 + q$$

Sketch the graph of $y = 24 - 2x - x^2$

$$y = 24 - 2x - x^2$$

a = -1 **2ap = -2** **$ap^2 + q = 24$**

p = 1 **-1 + q = 24**

q = 25

$$y = -(x+1)^2 + 25$$

Solution:

1. Write the equation in completed square form.

2. Establish turning point.

So minimum tp occurs at:

(-1 , 25)

3. Find y-intercept (X = 0)

$$y(0) = 24 - 2(0) - (0)^2 = 24$$

Y-Intercept is (0, 24)

Example 3 (Negatives)

Sketch the graph of $y = 24 - 2x - x^2$

Solution:

4. Find Roots (Y = 0)

$$y = 24 - 2x - x^2$$

$$y = -(x+1)^2 + 25$$

$$\Rightarrow -(x+1)^2 + 25 = 0$$

$$\Rightarrow -(x+1)^2 = -25$$

$$\Rightarrow (x+1)^2 = 25$$

$$\Rightarrow (x+1) = \pm 5$$

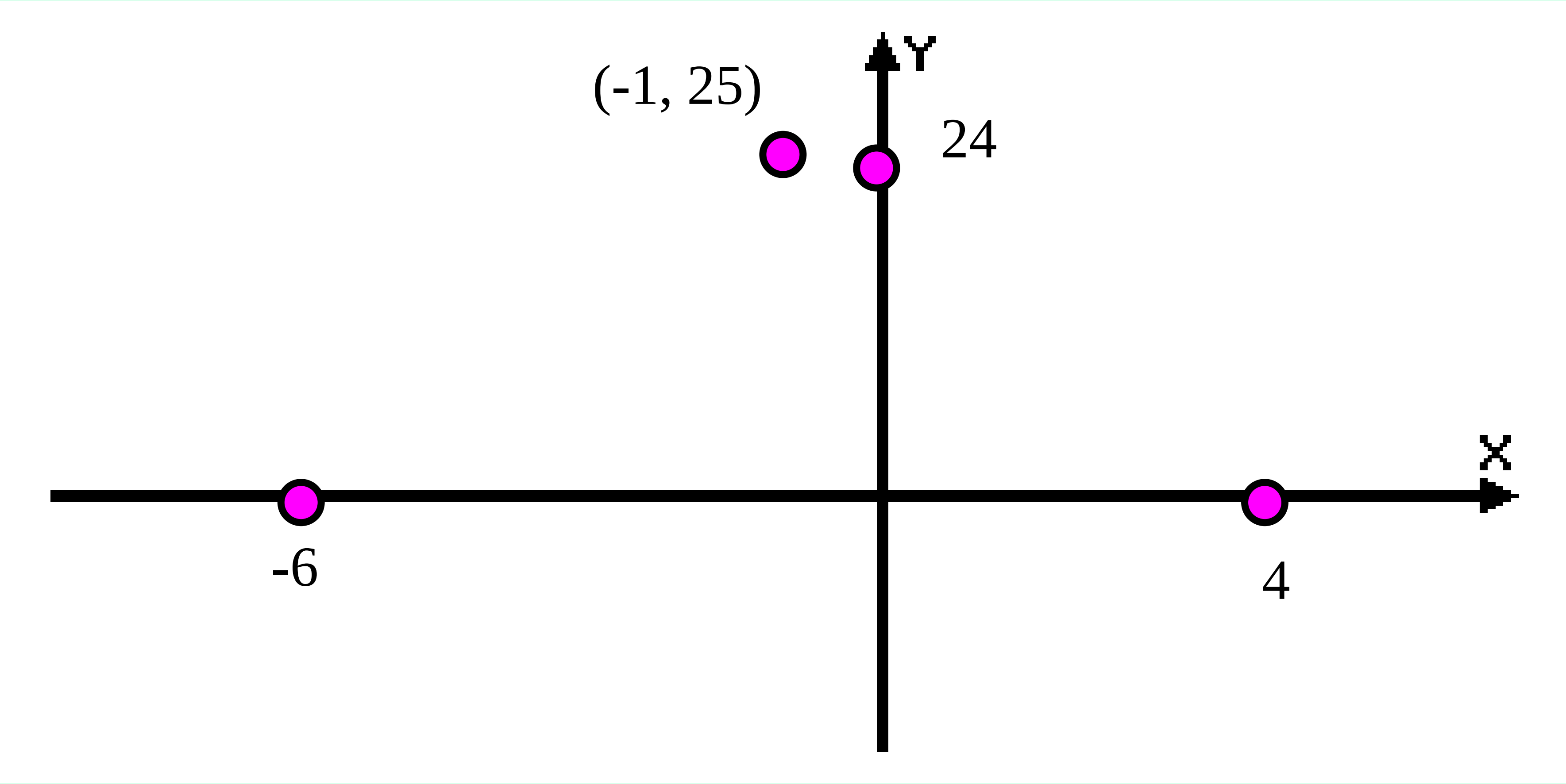
$$\Rightarrow x = 5 - 1 \quad or \quad x = -5 - 1$$

$$\Rightarrow x = 4 \quad or \quad x = -6$$

So Roots are:

(4 , 0) (-6 , 0)

5. Now use this information to plot points and join to make a parabola.



So maximum tp occurs at:

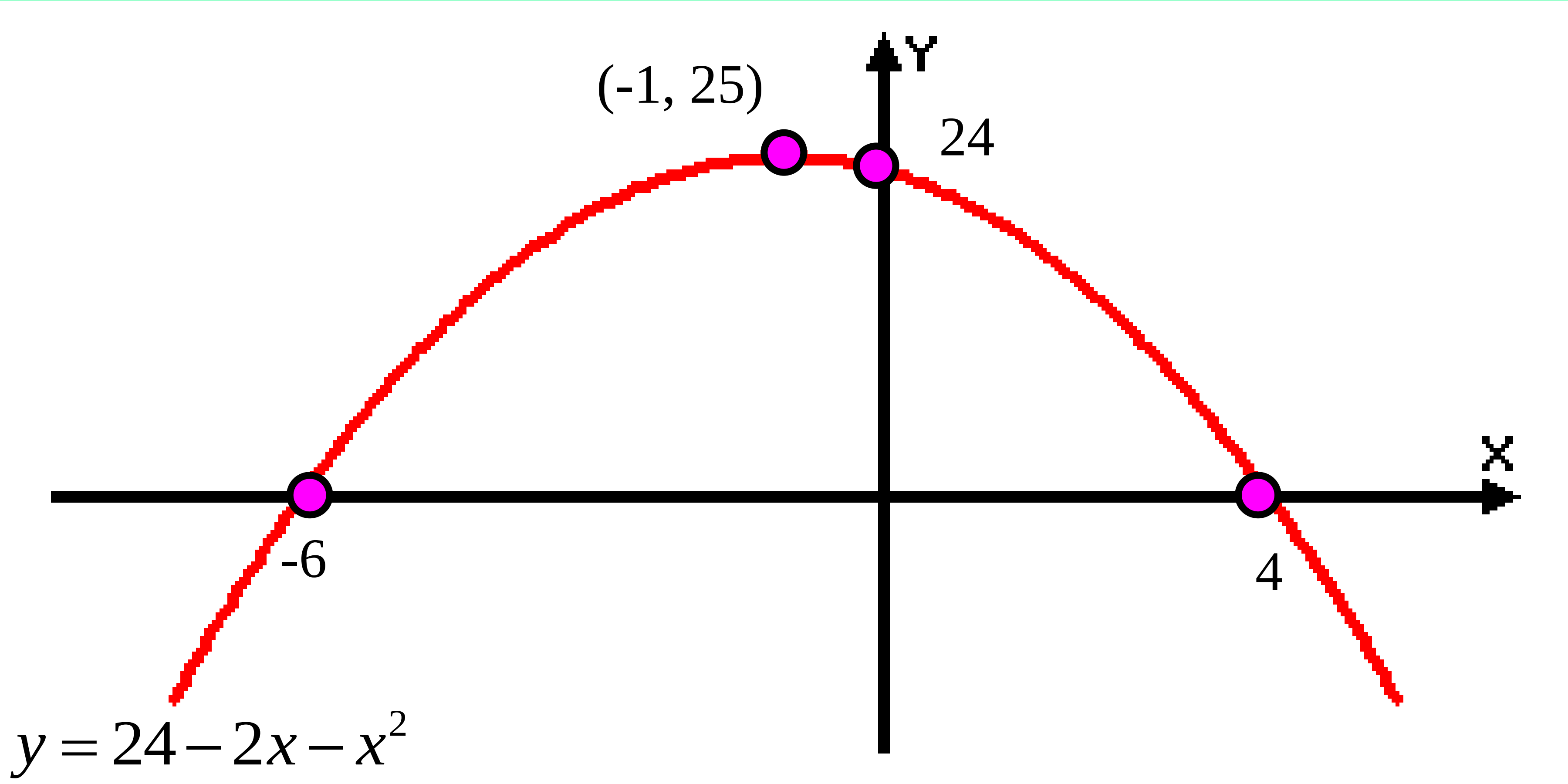
(-1 , 25)

Y-Intercept is (0, 24)

So Roots are:

(4 , 0)

(-6 , 0)



So maximum tp occurs at:

(-1 , 25)

Y-Intercept is (0, 24)

So Roots are:

(4 , 0)

(-6 , 0)

Heinemann, p.146, EX 8D,
Q2, (b) & (d)