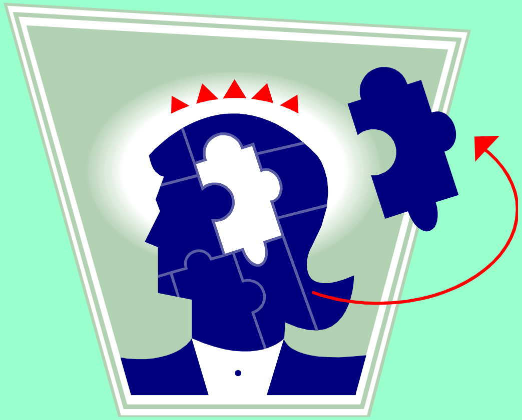
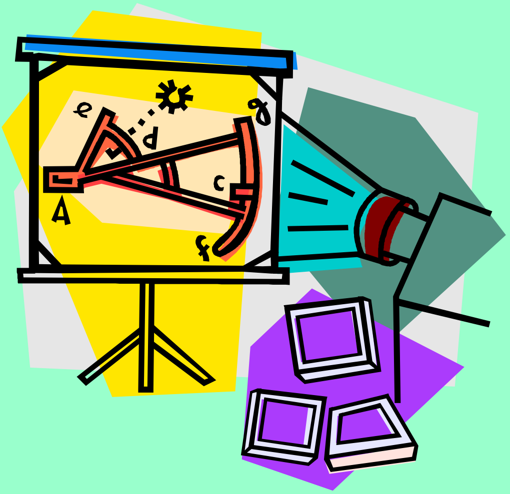


4.

Position vectors

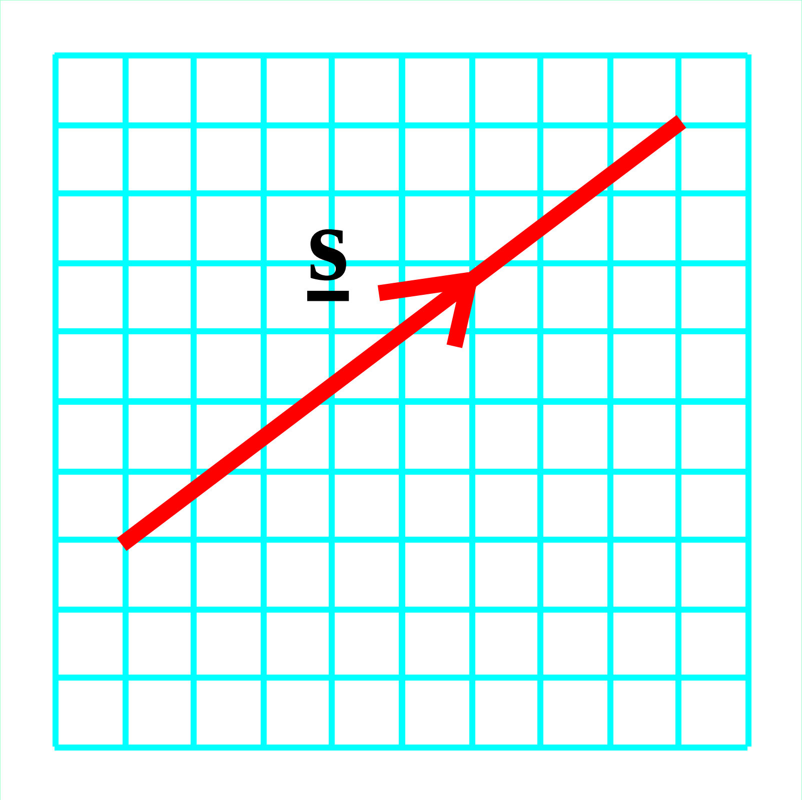
Including Unit vectors

and midpoints



Unit Vectors

For every vector there exists a parallel vector which **has a magnitude of exactly 1.** This is known as a unit vector.



Take the vector **s.**

In component form:

$$\underline{s} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

and so has magnitude

$$|\underline{s}| = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

So if we want a parallel vector which has magnitude 1, we must divide all components by 10.

$$\underline{s} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

so unit vector is

$$\frac{1}{10} \times \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{8}{10} \\ \frac{6}{10} \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix}$$

PROOF: Lets show unit vector has magnitude 1:

$$\left|\underline{unit\ vector}\right| = \sqrt{\left(\tfrac{4}{5}\right)^2 + \left(\tfrac{3}{5}\right)^2} = \sqrt{\left(\tfrac{16}{25}\right) + \left(\tfrac{9}{25}\right)} = \sqrt{1} = 1$$

Heinemann, p.238, EX 13F, Q1

This is not the end

Position Vectors

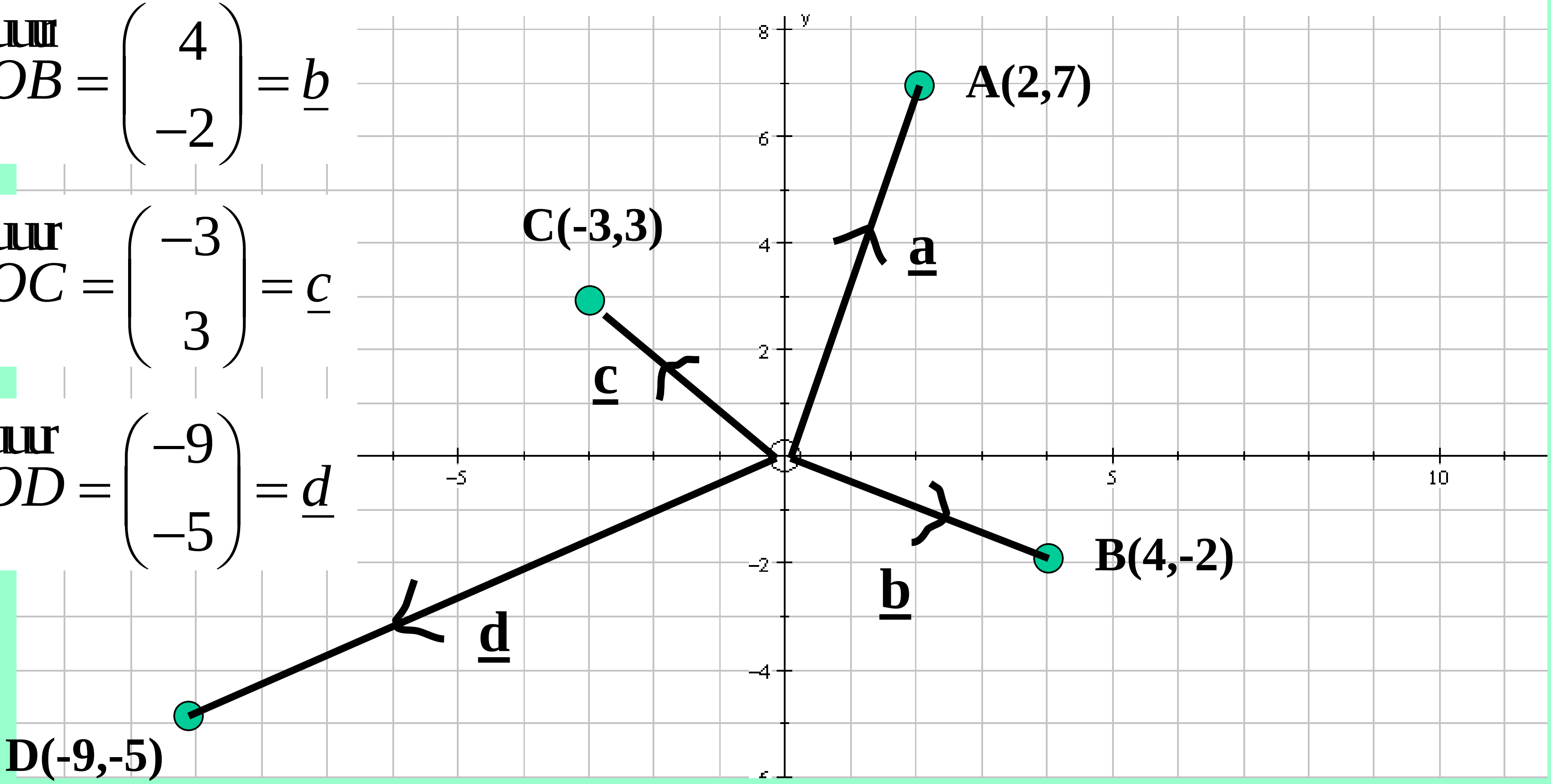
Position vectors are used to give points on a coordinate grid in component form relative to the origin.

Lets look at some examples:

$$\overrightarrow{OB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \underline{b}$$

$$\overrightarrow{OC} = \begin{pmatrix} -3 \\ 3 \end{pmatrix} = \underline{c}$$

$$\overrightarrow{OD} = \begin{pmatrix} -9 \\ -5 \end{pmatrix} = \underline{d}$$



A is (2,7)

$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} = \underline{a}$

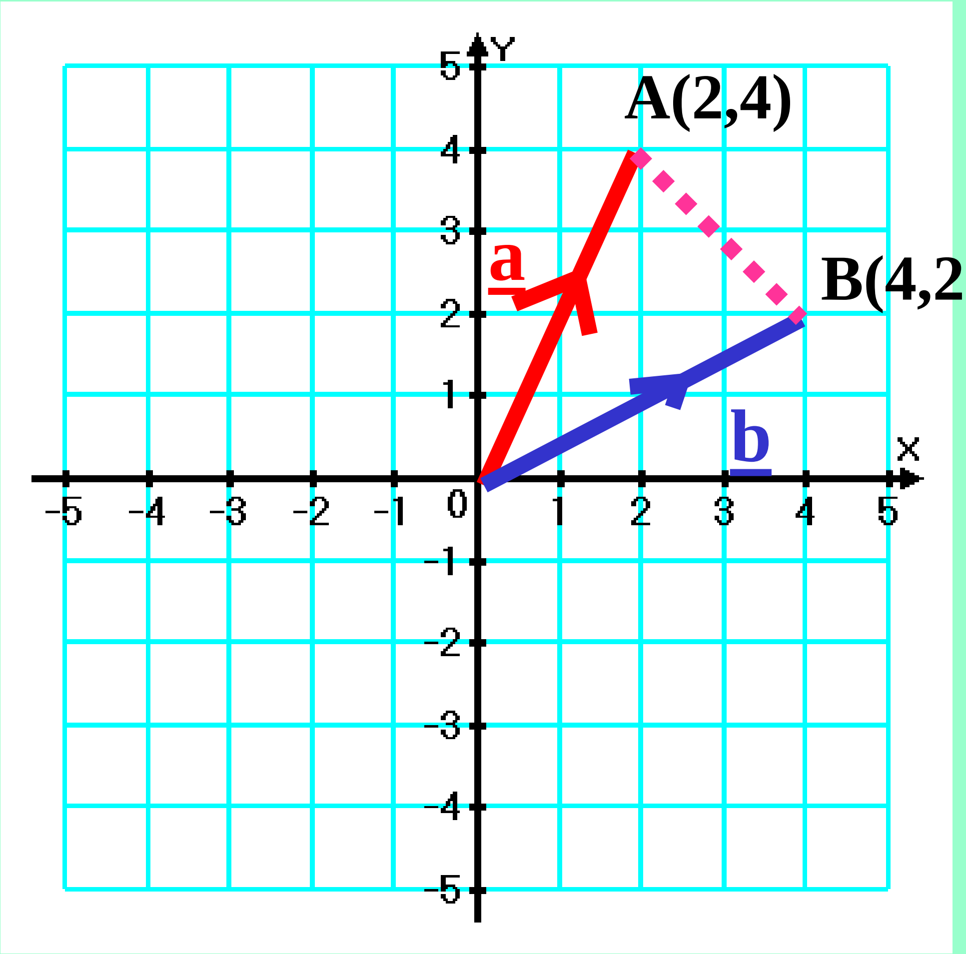
position vector

point

Directed line segment

components

Lets look now at how we perform additions on position vectors:



"nose to tail"

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

$$= -\underline{a} + \underline{b}$$

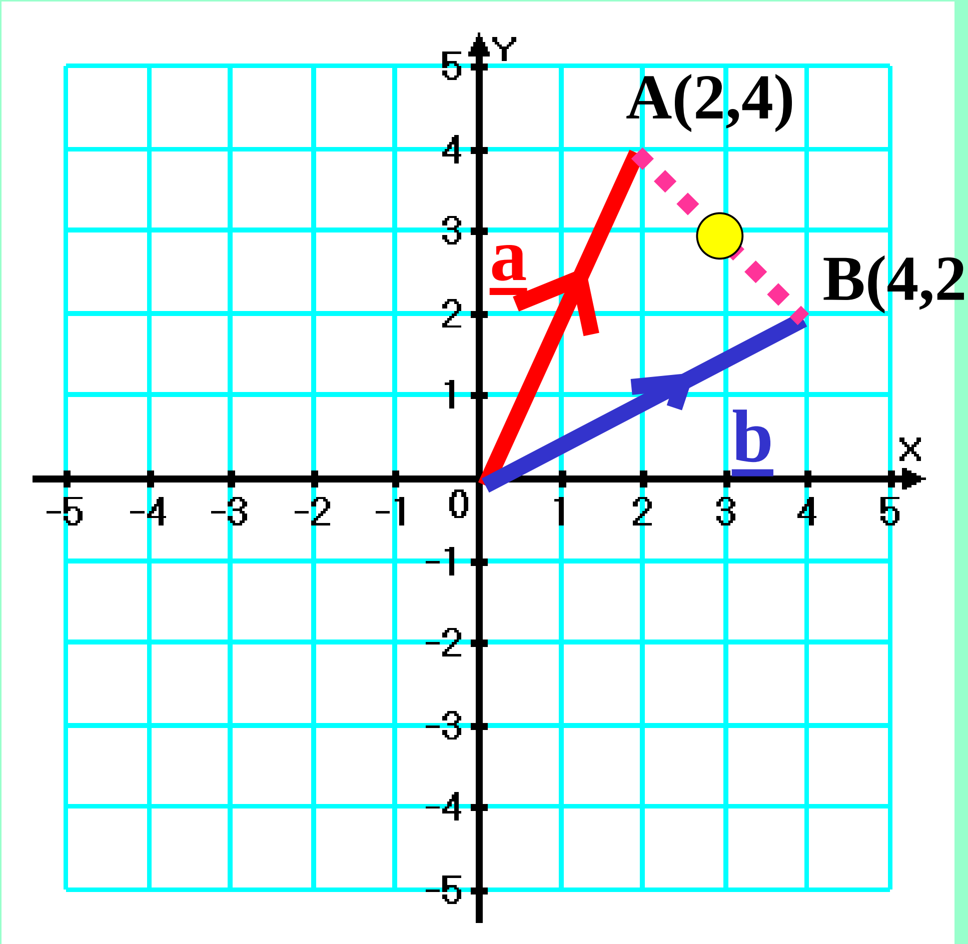
"2nd – 1st"

$$= \underline{b} - \underline{a}$$

$$= \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

How could we find the midpoint of $\overrightarrow{AB}$?



$$M = \tfrac{1}{2}(\underline{a} + \underline{b})$$

$$= \begin{pmatrix} \frac{2+4}{2} \\ \frac{4+2}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

So coordinates of midpoint are (3,3)

Here is one for you to try:

Example

(a) Find the components of $\overrightarrow{CD}$ if C is (-2,5) and D is (0,9)

(b) Find the midpoint of $\overrightarrow{CD}$ in component form.

"2nd – 1st"

Solution:

$$\overrightarrow{CD} = \underline{d} - \underline{c} = \begin{pmatrix} 0 \\ 9 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$M = \begin{pmatrix} \frac{-2+0}{2} \\ \frac{5+9}{2} \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$$

Coordinates of M are (-1,7)

Heinemann, p.239, EX 13G, ALL