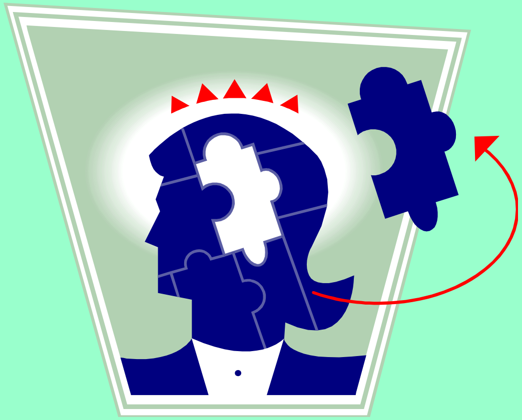
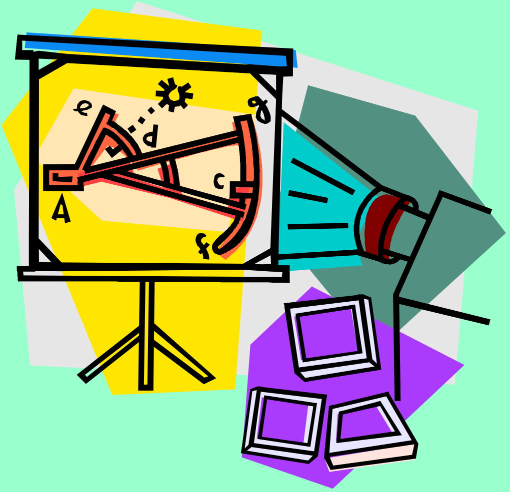


4. Integrating $(ax+b)^n$

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$



Integrating brackets

We saw when differentiating brackets that

$$\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1}$$

As both the original and the result contain (ax+b) it is reasonable to assume that the integral will contain $(ax+b)^{\mathbf{n+1}}$

Thus $\int (2x+3)^4 \, dx$ should include $(2x+3)^5$

However, if we differentiate the function $(2x + 3)^5$ we get:

$$\frac{dy}{dx} = 5(2x+3)^4 \times 2 = 10(2x+3)^4$$

This is 10 times bigger than what we want.

So: $\int (2x+3)^4 \, dx = \frac{(2x+3)^5}{10}$ ← **10 = 2 x 5**

When we differentiated, the power and the coefficient of ***x*** were involved in the derivative. A similar thing happens with the integral except this time we use the **NEW** power.

This leads to the following rule:

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$$

Example 1

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$$

Find $\int (3x+5)^3 \, dx$

Solution:

$$\int (3x+5)^3 \, dx = \frac{(3x+5)^4}{3\times 4} + C$$

$$= \frac{(3x+5)^4}{12} + C$$

Heinemann, p.274, EX 14J, Q2

This is not the end

Example 2

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$$

Find $\int \frac{dx}{(4x+5)^2}$

Solution:

$$\int \frac{dx}{(4x+5)^2} = \int (4x+5)^{-2} \, dx$$

$$= \frac{(4x+5)^{-1}}{4 \times -1} + C$$

$$= -\frac{(4x+5)^{-1}}{4} + C$$

$$= -\frac{1}{4(4x+5)} + C$$

Heinemann, p.274, EX 14J, Q3(b) &(d)

This is not the end

Example 3

NAB

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$$

Find $\displaystyle\int_1^2 (2x-1)^3 \, dx$

Solution:

$$\int_1^2 (2x-1)^3 \, dx = \left[\frac{(2x-1)^4}{4\times 2}\right]_1^2$$

$$= \left[\frac{(2x-1)^4}{8}\right]_1^2$$

$$= \left[\frac{(2\times 2-1)^4}{8}\right] - \left[\frac{(2\times 1-1)^4}{8}\right]$$

Example 3

NAB

Find $\int_1^2 (2x-1)^3 \, dx$

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$$

Solution:

$$\int_1^2 (2x-1)^3 \, dx = \left[\frac{(3)^4}{8}\right] - \left[\frac{(1)^4}{8}\right]$$

$$= \left[\frac{81}{8}\right] - \left[\frac{1}{8}\right]$$

$$= \left[\frac{80}{8}\right]$$

= 10

Heinemann, p.274, EX 14J,Q4 & 5