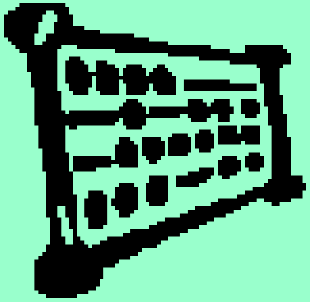
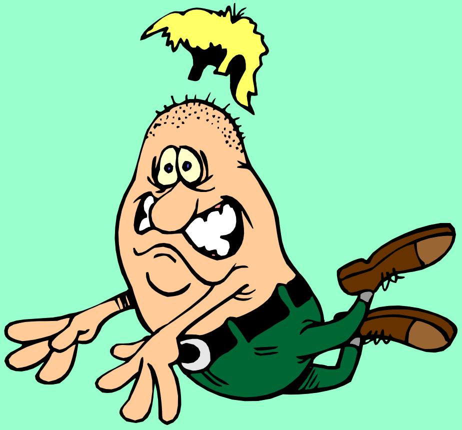
3.

Factor Theorem

If f(h) = 0 then (x - h) is a factor of f(x)





A numerical example of factors

The factors of 24 are: 1 x 24

2 x 12

3 x 8

4 x 6

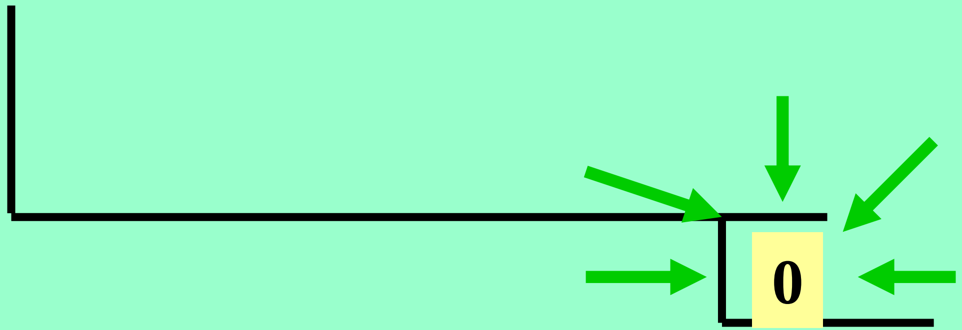
If we divided 24 by any of these factors there would be no remainder.

This can be expressed as: $f(h) = 0 \Leftrightarrow (x - h)$ is a factor of f(x)

x^4 x^3 x^2 x x^0

In terms of synthetic division this means there must be no remainder:

x = **h**



Example 1

NAB

Show that (x + 2)is a factor of $f(x) = x^3 + 4x^2 - 11x - 30$ and express f(x) in fully factorised form.

Solution:

x +2 = 0 → x = -2

	x^3	x^2	x	x^0
-2	1	4	-11	-30
		-2	-4	30
	1	2	-15	0

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-15)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{64}}{2}$$

$x = 3$ or -5

As f(-2) = 0, (x + 2) is a factor.

So $f(x) = (x+2)(x^2+2x-15)$

So $f(x) = (x+2)(x-3)(x+5)$

Example 2

Show that $(2x + 1)$ is a factor of $f(x) = 2x^3 + 3x^2 - 3x - 2$

Solution:

$2x + 1 = 0$

$2x = -1$

	x^3	x^2	x	x^0
$x = -\frac{1}{2}$	2	3	-3	-2
		-1	-1	2
	2	2	-4	0

As $f(-\frac{1}{2}) = 0$, $(2x + 1)$ is a factor.

We want $(2x + 1)$ → $f(x) = (x + \frac{1}{2})(2x^2 + 2x - 4)$

X 2   Divide by 2

So $f(x) = (2x+1)(x^2 + x - 2)$

Example 3

Show that $(3x - 2)$ is a factor of $f(x) = 3x^3 - 5x^2 + 8x - 4$

Solution:

$3x - 2 = 0$

$3x = 2$

$x = 2/3$

	x^3	x^2	x	x^0
2/3	3	-5	8	-4
		2	-2	4
	3	-3	6	0

As $f(2/3) = 0$, $(3x - 2)$ is a factor.

We want $(3x - 2)$ → $f(x) = (x - 2/3)(3x^2 - 3x + 6)$

X 3 Divide by 3

So $f(x) = (3x - 2)(x^2 - x + 2)$

Heinemann, p.131, EX 7E, Q1 to 5 then EX7C, Q4(a) to (d)