

**3.**

# Gradients

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

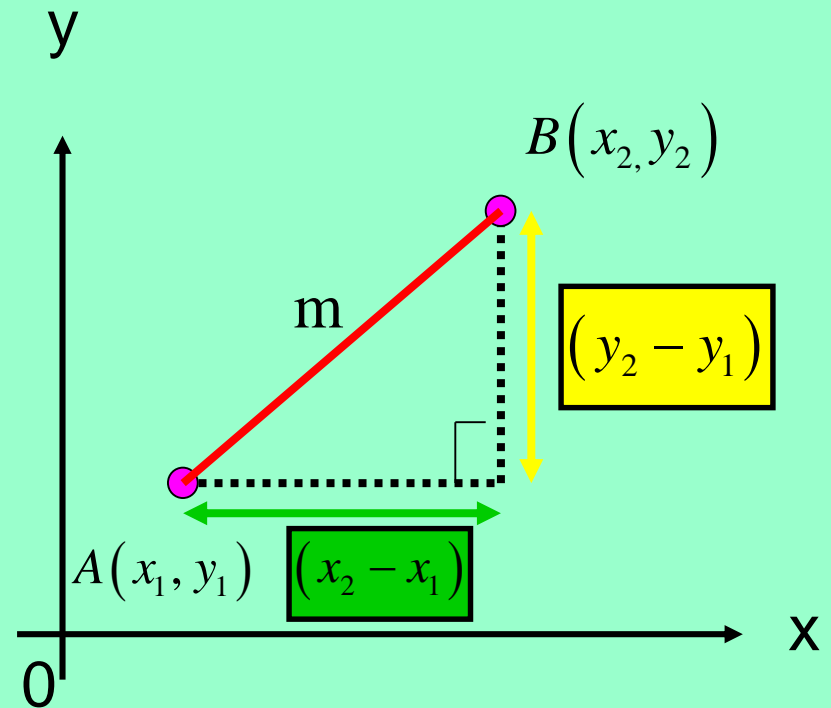
$$m = \tan \theta$$

## The Gradient Formula

We have two points on the coordinate grid, A and B.

We want to find the steepness, or gradient of the line joining these points.

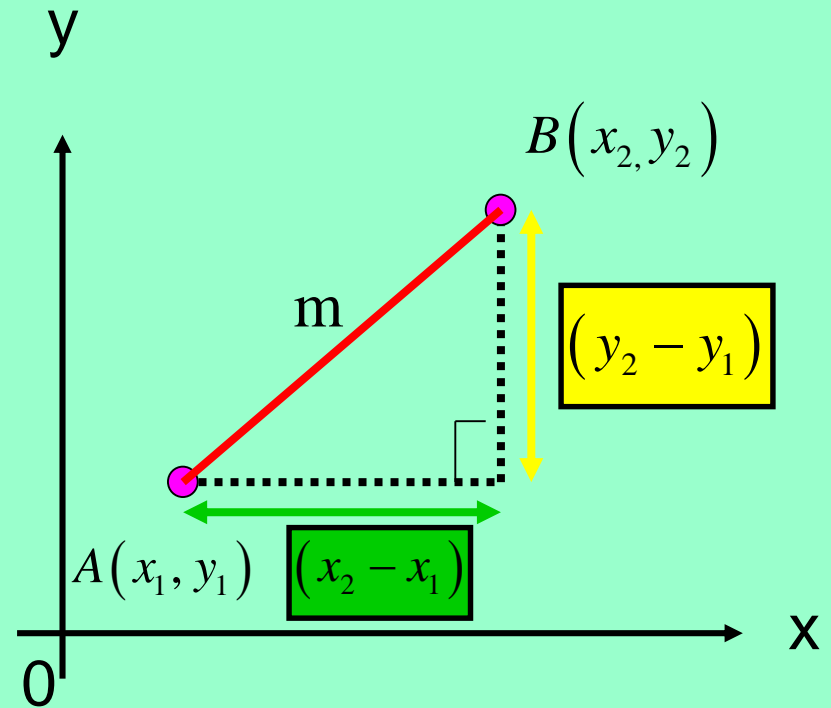
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## The Gradient Formula

We know from previous work that:

$$\begin{aligned} \text{Gradient} &= \frac{y - \text{change}}{x - \text{change}} \\ &= \frac{(y_2 - y_1)}{(x_2 - x_1)} \end{aligned}$$



# The Gradient Formula

## Example 1

Calculate the gradient of the lines through the following points:

- (a) (3, 7) and (5, 13)
- (b) (6, 2) and (3, 8)
- (c) (-3, 4) and (3, 4)
- (d) (-2, -3) and (-2, 9)

## Solution:

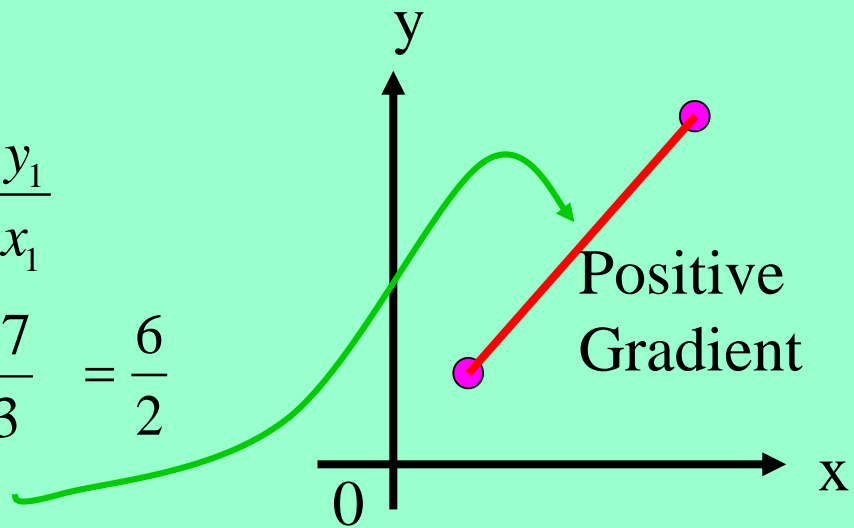
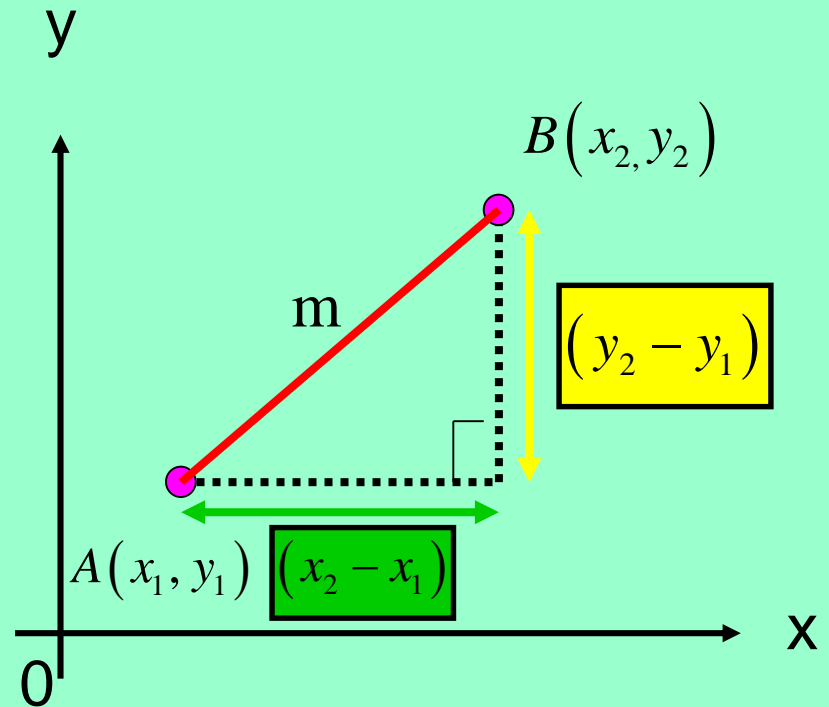
- (a) (3, 7) and (5, 13)

$(x_1, y_1)$	$(x_2, y_2)$
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$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{13 - 7}{5 - 3} = \frac{6}{2}$$

$$m = 3$$



## Solution (cont.) :

(b) (6 , 2) and (3 , 8)

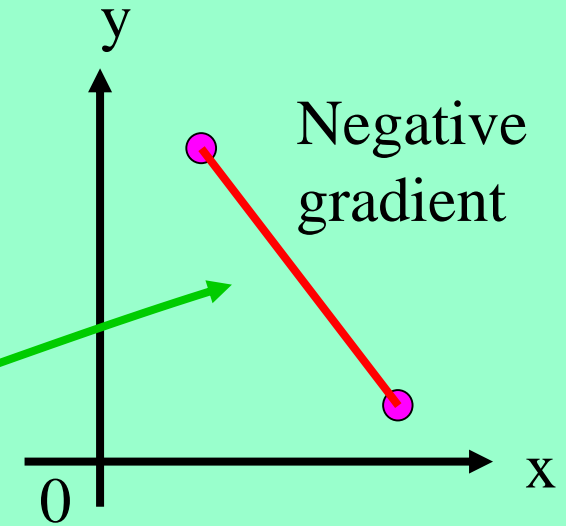
$(x_1, y_1)$

$(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{8 - 2}{3 - 6} = \frac{6}{-3}$$

$$m = -2$$



(c) (-3 , 4) and (3 , 4)

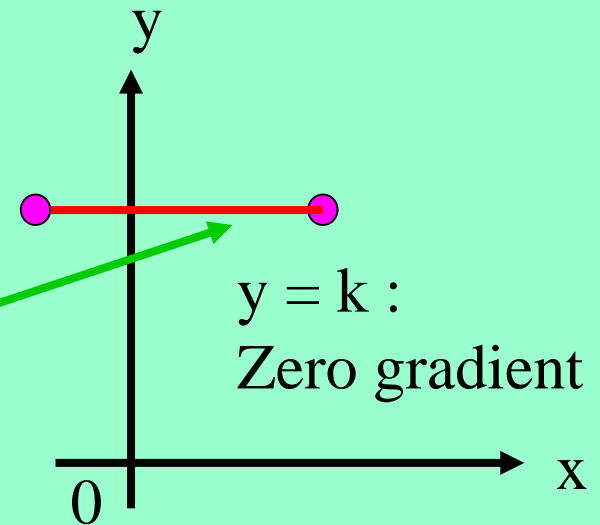
$(x_1, y_1)$

$(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4 - 4}{3 - (-3)} = \frac{0}{6}$$

$$m = 0$$



Horizontal Lines have Zero gradient

## Solution (cont.) :

(d)  $(-2, -3)$  and  $(-2, 9)$

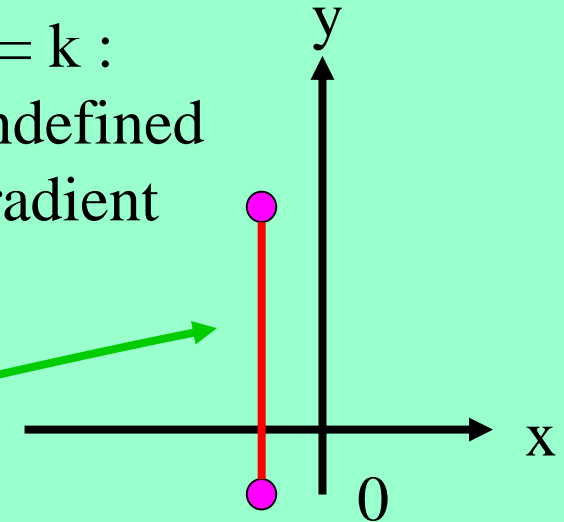
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{9 - (-3)}{-2 - (-2)} = \frac{12}{0}$$

$(x_1, y_1)$

$(x_2, y_2)$

$$m = \infty$$

$x = k$  :  
undefined  
gradient



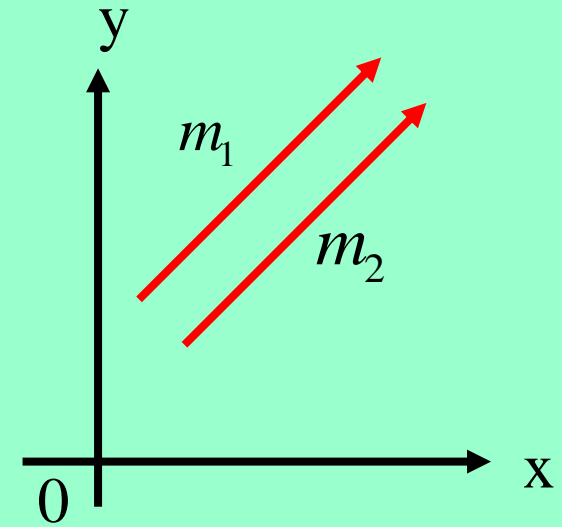
Vertical Lines have undefined gradient

Heinemann , p.4, EX 1B, Q 4

## Parallel Lines

Parallel lines have the same gradient

$m_1 = m_2 \Leftrightarrow$  lines are parallel



Heinemann , p.2, EX 1A Q5 &8

## The Angle Made with the $x$ -axis

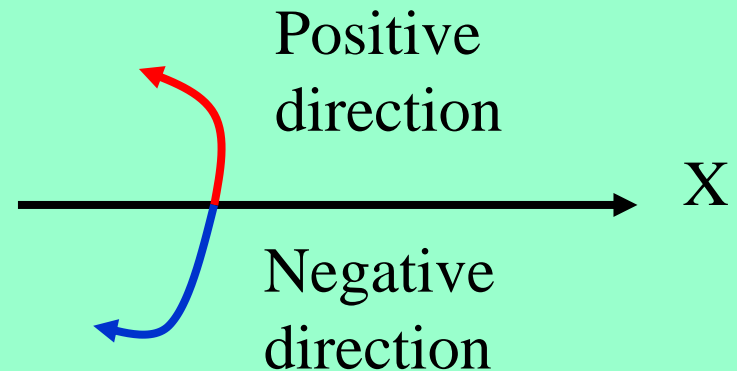
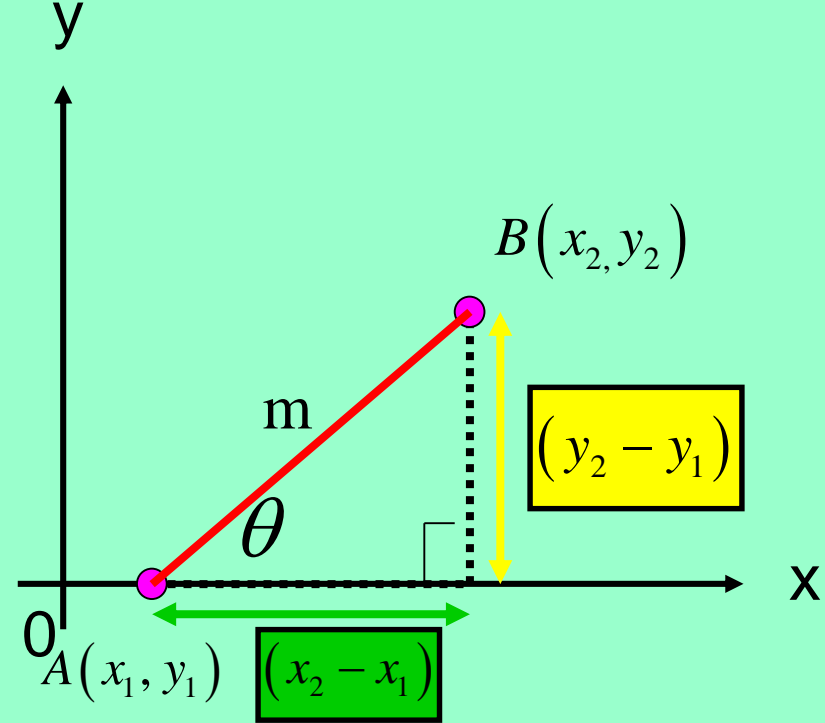
Some questions require us to calculate the angle a straight line makes with the  $x$ -axis. (this is known as “theta”).

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From SOH CAH TOA :

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \textit{Gradient}$$

$$\Rightarrow m = \tan \theta$$





## Example 1

Find the size of the angle between the positive x-axis and a line with gradient  $\sqrt{3}$

**Solution:**

$$m = \tan \theta$$

$$\tan \theta = \sqrt{3}$$

$$\theta = \tan^{-1}(\sqrt{3})$$

$$\theta = 60^{\circ}$$

## Example 2

Find the angle between line AB and the positive x-axis given that A is (4, -1) and B is (8, 5).

### Solution:

1. Find the gradient of the line

2. Use this gradient and  $m = \tan \theta$  to find angle.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-1)}{8 - 4}$$

$$m = \frac{6}{4} = \frac{3}{2}$$

$$\tan \theta = \frac{3}{2}$$

$$\theta = \tan^{-1} \left( \frac{3}{2} \right)$$

$$\theta = 56.3^\circ$$

### Example 3

NAB

Find the gradient of the line inclined to the positive x-axis at an angle of  $135^{\circ}$

Solution:

$$m = \tan \theta$$

$$m = \tan 135^{\circ}$$

$$m = -1$$

Heinemann , p.4, EX 1B, Q6