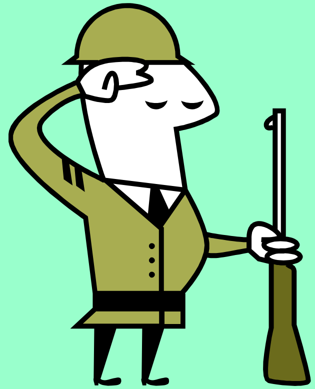


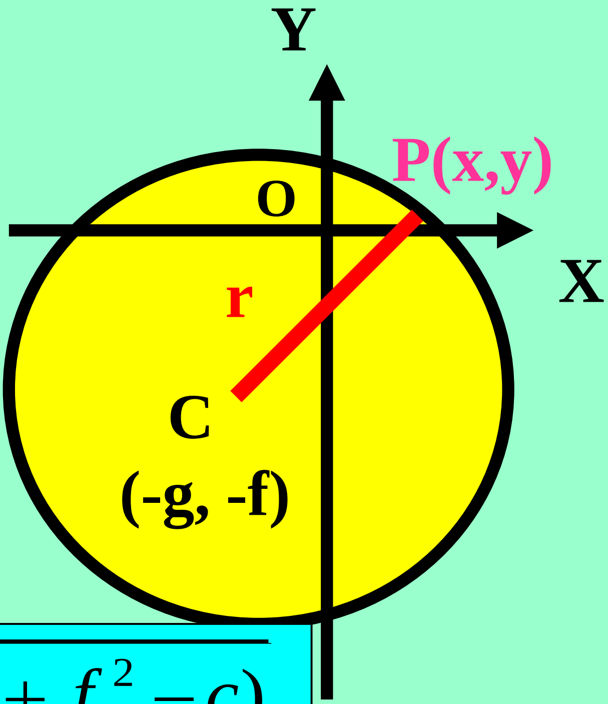
3.

The General Equation of a Circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$r = \sqrt{(g^2 + f^2 - c)}$$

General Equation of a circle



Take a circle with centre (-g, -f).

The general equation for a circle is:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

where the **centre is (-g , -f)** and $r = \sqrt{(g^2 + f^2 - c)}$

Proof

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$x^2 + 2gx + y^2 + 2fy = -c$$

$$(x^2 + 2gx + g^2) + (y^2 + 2fy + f^2) = g^2 + f^2 - c$$

$$(x + g)^2 + (y + f)^2 = g^2 + f^2 - c$$

This is now in the form $(x-a)^2 + (y-b)^2 = r^2$

$(a,b) = (-g, -f)$

$r^2 = g^2 + f^2 - c$

Example

Find the **general equation** of a circle with a centre (1 , 2) and r = 4

Solution:

$$(x-a)^2+(y-b)^2=r^2$$

$$(x-1)^2+(y-2)^2=4^2$$

$$(x^2-2x+1)+(y^2-4y+4)=16$$

$$x^2+y^2-2x-4y+5=16$$

$$x^2+y^2-2x-4y-11=0$$

Note that for centre: x & y coords doubled and sign changed.

General Equation of a Circle

Copy the following:

The equation of any circle with centre **C (-g, -f)** is:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

The radius can be found from: $r = \sqrt{(g^2 + f^2 - c)}$

NB : $g^2 + f^2 - c$ must be **POSITIVE** or we do not have a circle.

Example 1

NAB

Identify g, f and c for this circle and hence write down the centre and the radius: $x^2 + y^2 - 4x - 6y + 9 = 0$

Solution:

Compare to general equation:

$$x^2 + y^2 - 4x - 6y + 9 = 0$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

2g = -4   2f = -6

So g = -2 and f = -3 and c = 9

Centre is C(2 , 3)

Radius = 2 units

$$r = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{(-2)^2 + (-3)^2 - 9}$$

$$= \sqrt{4 + 9 - 9}$$

= 2

Example 2

The point (2, -3) lies on the circumference of the circle

$x^2 + y^2 - 4x - 8y + c = 0$ Find the value of c.

Solution:

All points conform to equation so sub x = 2, y = -3 into equation:

$$x^2 + y^2 - 4x - 8y + c = 0$$

$$(2)^2 + (-3)^2 - 4(2) - 8(-3) + c = 0$$

$$4 + 9 - 8 + 24 + c = 0$$

$$29 + c = 0$$

$$c = -29$$

Example 3

Find the **general equation** of a circle with centre (-8, -15) if the circumference passes through the origin.

Solution:

NB : if the circumference of a circle passes through the origin there is no constant term **i.e. c = 0**

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

For g = 8, f = 15 and c = 0 :

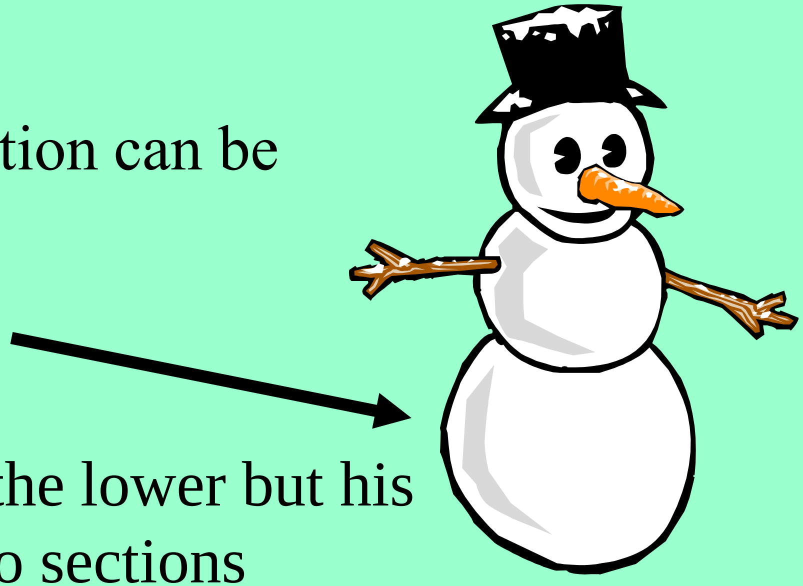
$$x^2 + y^2 + 2(8)x + 2(15)y + 0 = 0$$

$$x^2 + y^2 + 16x + 30y = 0$$

Example 4

Frosty the Snowman's lower body section can be represented by the equation

$$x^2 + y^2 - 6x + 2y - 26 = 0$$



His middle section is the same size as the lower but his head is only $^1/_3$ the size of the other two sections

Find the equation of his head !

Solution:

For equation must have C and r

Start with lower body:

$$x^2 + y^2 - 6x + 2y - 26 = 0$$

Compare to general equation:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

2g = -6

2f = 2

So g = -3 and f = 1 and c = -26

$$r = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{(-3)^2 + (1)^2 - (-26)}$$

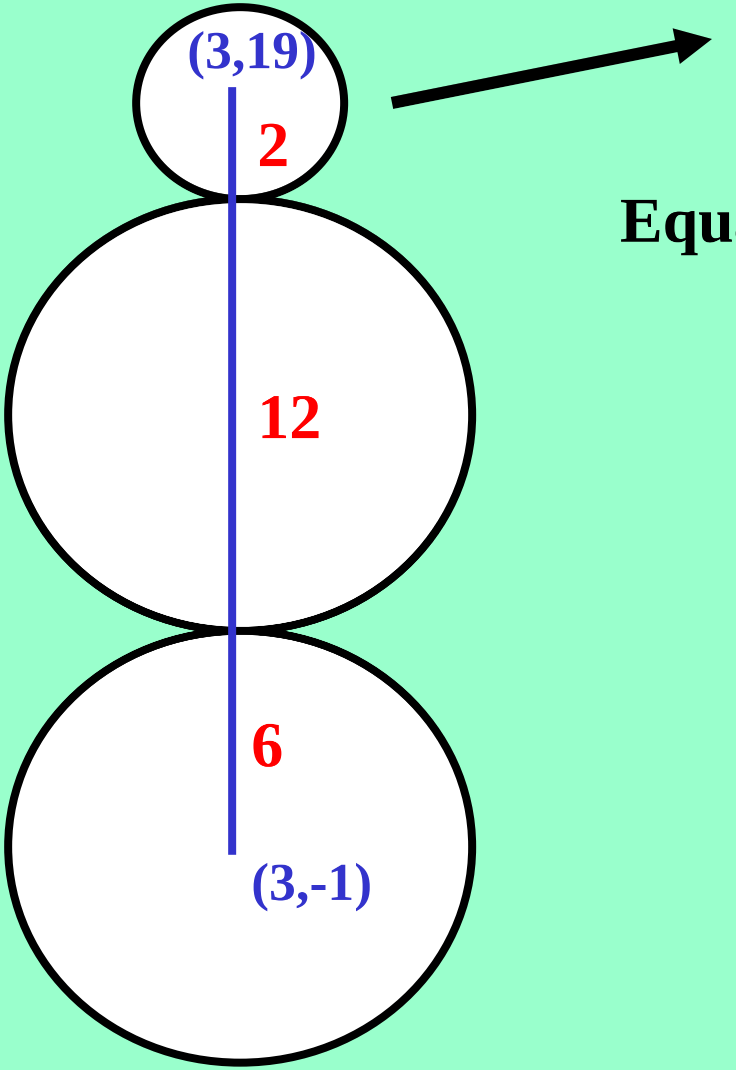
$$= \sqrt{9 + 1 + 26}$$

= 6

Centre is C(3 , -1)

Radius = 6 units

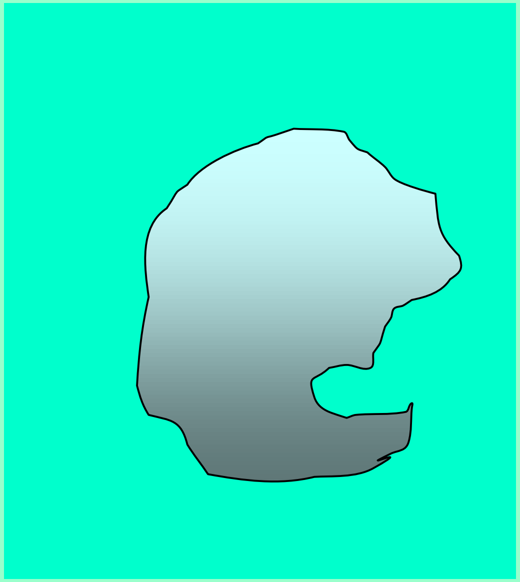
From question radius of middle body = 6 and radius of head = 2



Using $(x-a)^2+(y-b)^2=r^2$

Equation is $(x-3)^2+(y-19)^2=4$

Ha ha ha !



Heinemann,
p.213, EX 12H, Q1 (RH Col),
Q 2, 3, & 4