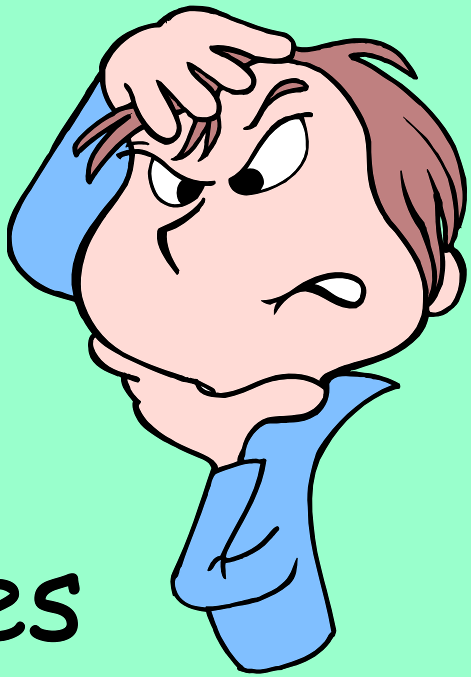


3.

Trig Exact Values

Exact Values

Sometimes it helps us when manipulating and calculating things if they are in a “user friendly” form.

Sometimes in the middle of a calculation, rather than rounding, we want to maintain the accuracy or **exact value**.

In trigonometry there are certain angles which lend themselves to easy manipulation and maintaining accuracy by keeping them as a ratio (i.e. an exact value).

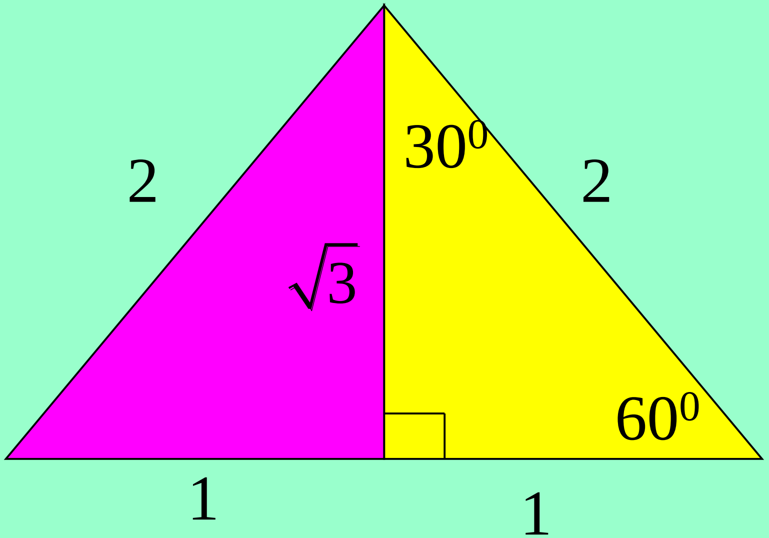
Just as with radians, these angles are 30^0, 45^0, 60^0 and 90^0.

Lets look at two special triangles containing only these angles:

Exact Values

Copy the following:

Equilateral triangle – side 2 units



By pythag: height = $\sqrt{2^2 - 1^2} = \sqrt{3}$

$$\sin 30^0 = \frac{opp}{hyp} = \frac{1}{2}$$

$$\cos 30^0 = \frac{adj}{hyp} = \frac{\sqrt{3}}{2}$$

$$\tan 30^0 = \frac{opp}{adj} = \frac{1}{\sqrt{3}}$$

$$\sin 60^0 = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}$$

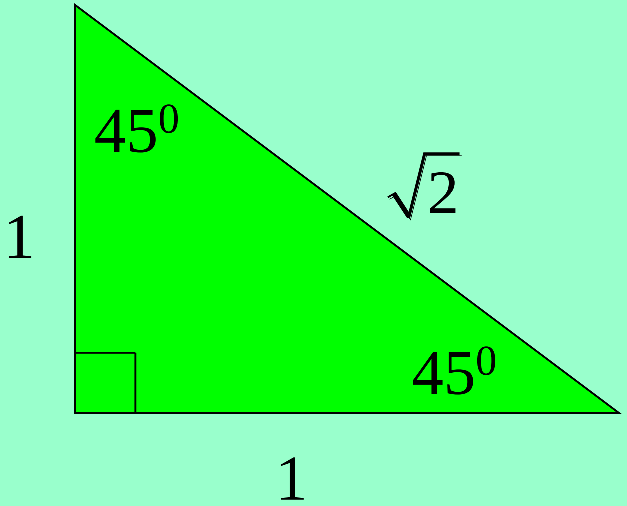
$$\cos 60^0 = \frac{adj}{hyp} = \frac{1}{2}$$

$$\tan 60^0 = \frac{opp}{adj} = \frac{\sqrt{3}}{1}$$

Exact Values

Copy the following:

Isosceles triangle – side 1 unit



$$\sin 45^0 = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$$

$$\cos 45^0 = \frac{adj}{hyp} = \frac{1}{\sqrt{2}}$$

$$\tan 45^0 = \frac{opp}{adj} = \frac{1}{1} = 1$$

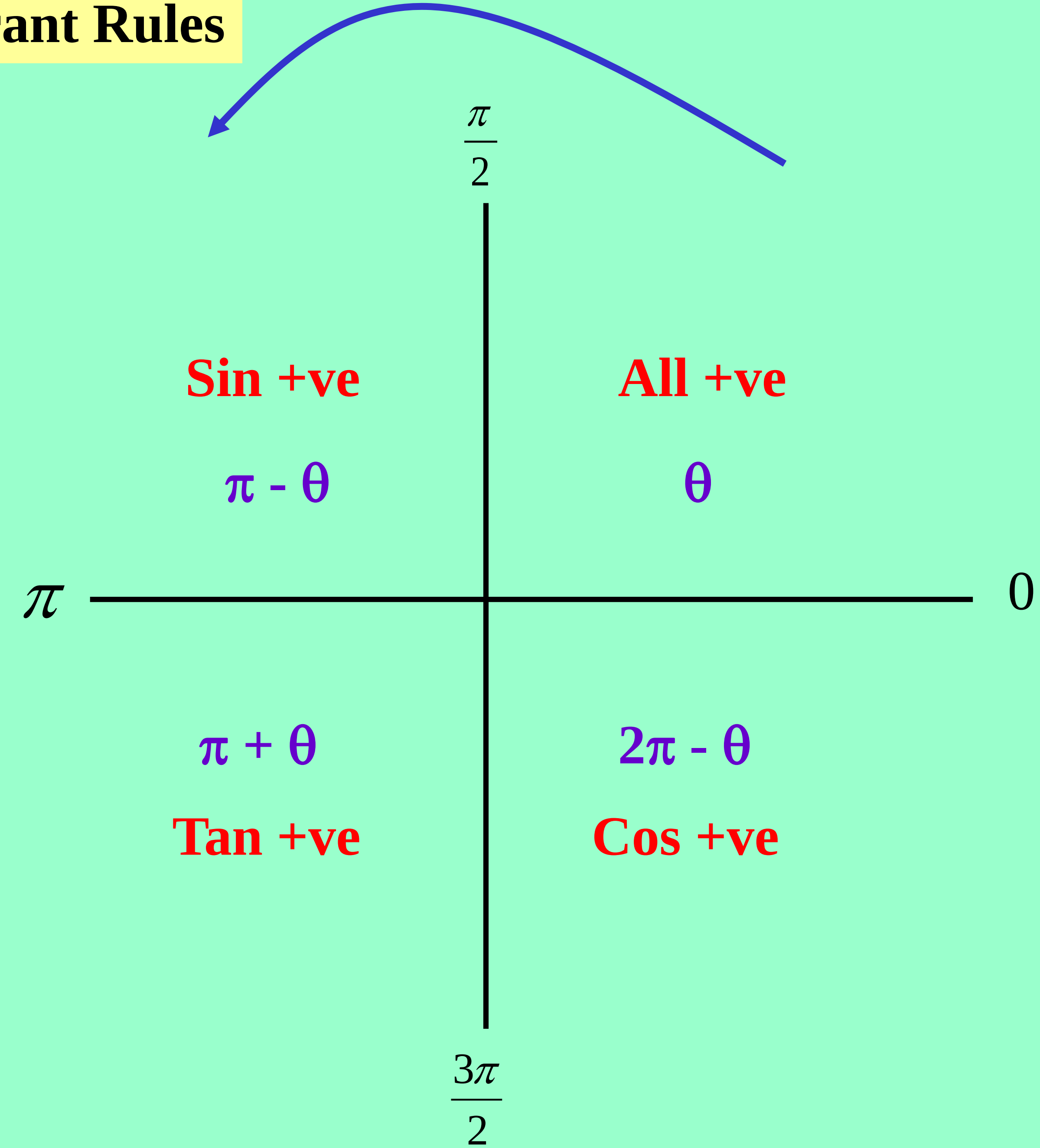
By pythag: hyp = $\sqrt{1^2 + 1^2} = \sqrt{2}$

This leads to the following table:

Exact Values

	0^0	30^0	45^0	60^0	90^0
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Und.

Quadrant Rules



Example 1

Find the exact value of :

(a) $\cos 240^0$ ← Q3

$= \cos (180 + 60)^0$

$= -\cos (60^0)$

$= -\frac{1}{2}$

(b) $\sin (-135^0)$

$= \sin 225^0$ ← Q3

$= \sin (180 + 45)^0$

$= -\sin (45^0)$

$= -\frac{1}{\sqrt{2}}$

(c) $\tan \frac{5\pi}{6}$

Q2

$= \tan 150^0$

$= -\tan (180 - 150)^0$

$= -\tan (30^0)$

$= -\frac{1}{\sqrt{3}}$

Add P= 360^0

Example 2

Find the exact value of $3 \sin^2 225^0 \cos 240^0$

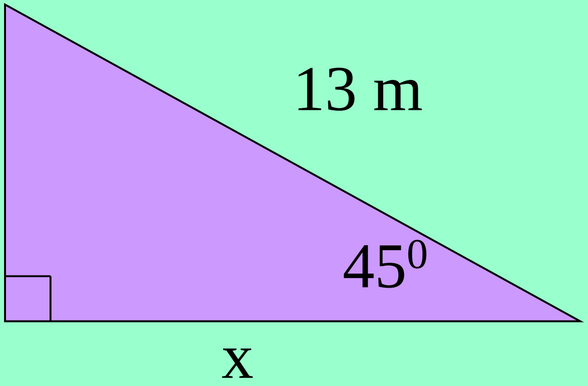
$$= 3 \times \left(-\frac{1}{\sqrt{2}}\right)^2 \times \left(-\frac{1}{2}\right)$$

$$= \frac{3}{1} \times \frac{1}{2} \times -\frac{1}{2}$$

$$= -\frac{3}{4}$$

Example 3

Find the exact length of x.



Solution:

1. Using SOH CAH TOA write down ratio of what we know.

$$\cos 45^0 = \frac{x}{13}$$

2. Cross multiply and make x the subject

$$x = 13 \times \cos 45^0$$

$$x = 13 \times \frac{1}{\sqrt{2}}$$

3. Tidy up

$$x = \frac{13}{\sqrt{2}} \quad \times \frac{\sqrt{2}}{\sqrt{2}}$$

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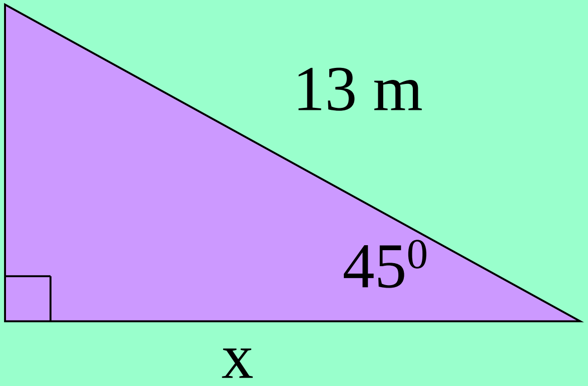
Find the exact length of x.



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$$x = \frac{13}{\sqrt{2}} = \frac{13\sqrt{2}}{2}$$

Heinemann, p.59, EX 4E
Q1 & 3