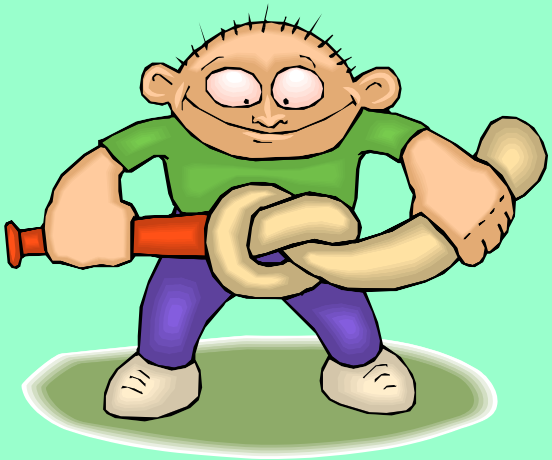
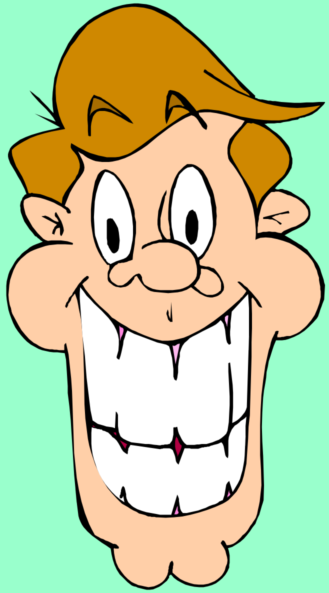


3. Double Angle Formulae



Double Angle Formulae

By modifying addition equations 1(a) and 2(a) we get the following "double angle" equations .....

3. $\sin 2A = 2\sin A\cos A$

4. $\cos 2A = \cos^2 A - \sin^2 A$

   or $2\cos^2 A - 1$

   or $1 - 2\sin^2 A$

These are on the formulae sheet.

We now look at why …

Recall that $\sin^2 A + \cos^2 A = 1$

So $\sin^2 A = 1 - \cos^2 A$ and $\cos^2 A = 1 - \sin^2 A$

$\sin 2A = \sin(A + A) = \sin A\cos A + \cos A\sin A = 2\sin A\cos A$

$\cos 2A = \cos(A + A) = \cos A\cos A - \sin A\sin A$

$= \cos^2 A - \sin^2 A$ **(version 1)**

$= \cos^2 A - (1 - \cos^2 A)$

$= \cos^2 A - 1 + \cos^2 A$

$= 2\cos^2 A - 1$ **(version 2)**

$= 2(1 - \sin^2 A) - 1$

$= 2 - 2\sin^2 A - 1$

$= 1 - 2\sin^2 A$ **(version 3)**

Example 1

Simplify:

(a) $2\cos^2 15^0 - 1$ ← **$\cos 2A = 2\cos^2 A - 1$** **Rule 4. (version 2)**

$= \cos(2 \times 15^0)$

$= \cos 30^0$

$$= \frac{\sqrt{3}}{2}$$

Example 1

Simplify:

(b) $2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$ ← **sin 2A= 2sinAcosA** Rule 3

$$= \sin\left(2 \times \frac{\pi}{8}\right)$$

$$= \sin\frac{\pi}{4} = \sin 45^0$$

$$= \frac{1}{\sqrt{2}}$$

Example 2

Express sin 6H and cos 6H in terms of sin 3H and cos 3H

Solution:

A = 3H

(a) $\sin(6H) = 2\sin(3H)\cos(3H)$ ← **sin 2A = 2sinAcosA**

(b) $\cos(6H) = \cos^2(3H) - \sin^2(3H)$ **cos 2A = cos²A - sin²A**

$= 2\cos^2(3H) - 1$ **cos 2A = 2cos²A - 1**

$= 1 - 2\sin^2(3H)$ **cos 2A = 1 - 2sin²A**

Example 3

Express sin P in terms of $\frac{P}{2}$

Solution:

$$\sin\left(P\right) = 2\sin\left(\tfrac{P}{2}\right)\cos\left(\tfrac{P}{2}\right)$$

A = P/2

sin 2A= 2sinAcosA

Example 4

$$\cos A = \frac{4}{\sqrt{17}} \quad \text{where} \quad 0 < A < \tfrac{\pi}{2}$$

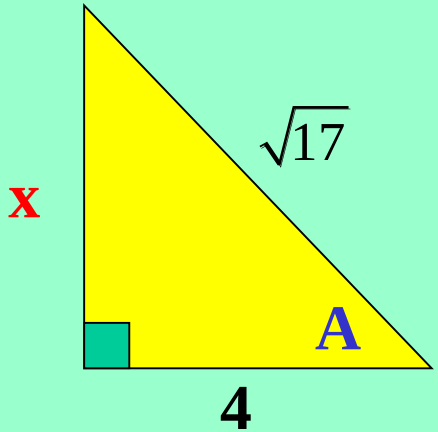
Calculate the exact values of sin(2A) and cos (2A)

Solution:

sin 2A= 2sinAcosA

need In quest.

$$\cos A = \frac{4}{\sqrt{17}} = \frac{adj.}{hyp.}$$



By pythag.

$$x^2 = \left(\sqrt{17}\right)^2 - 4^2$$

$$x^2 = 1$$

$$x = 1$$

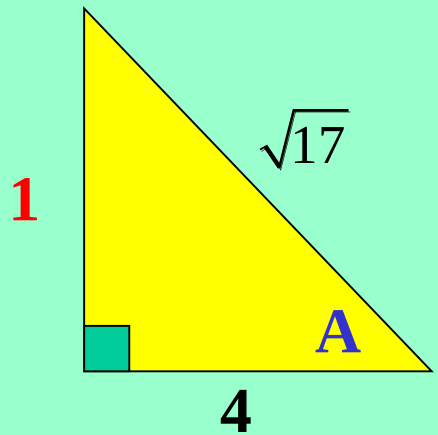
Example 4

$$\cos A = \frac{4}{\sqrt{17}} \quad \text{where} \quad 0 < A < \tfrac{\pi}{2}$$

Calculate the exact values of sin(2A) and cos (2A)

Solution:

$$\cos A = \frac{4}{\sqrt{17}} = \frac{adj.}{hyp.}$$



sin 2A= 2sinAcosA

$$\sin(2A) = 2 \times \left(\frac{1}{\sqrt{17}}\right) \times \left(\frac{4}{\sqrt{17}}\right)$$

$$\sin(2A) = \frac{8}{17}$$

Example 4

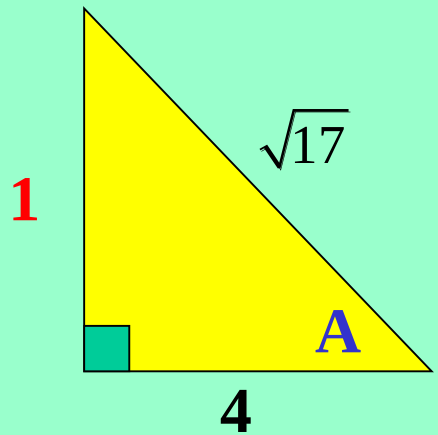
$$\cos A = \frac{4}{\sqrt{17}} \quad \text{where} \quad 0 < A < \tfrac{\pi}{2}$$

Calculate the exact values of sin(2A) and cos (2A)

Given cos A so use the version only involving cosA

Solution:

$$\cos A = \frac{4}{\sqrt{17}} = \frac{adj.}{hyp.}$$



cos 2A = 2cos²A - 1

$$\cos(2A) = 2 \times \left(\frac{4}{\sqrt{17}}\right)^2 - 1$$

$$\cos(2A) = 2 \times \left(\frac{16}{17}\right) - 1$$

$$\cos(2A) = \left(\frac{32}{17}\right) - \left(\frac{17}{17}\right)$$

$$\cos(2A) = \frac{15}{17}$$

Heinemann,
p.196, EX 11G, Q2, 3, 6, 7, 8,10,11