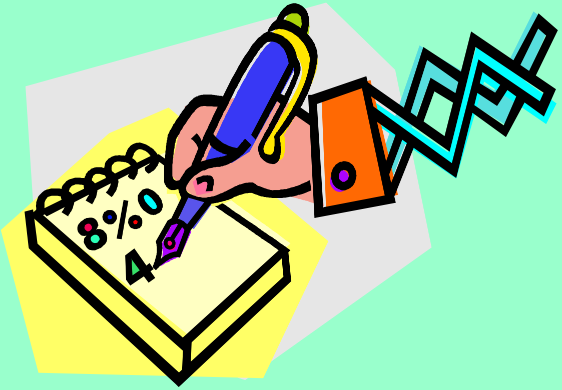
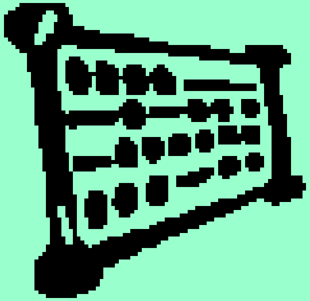
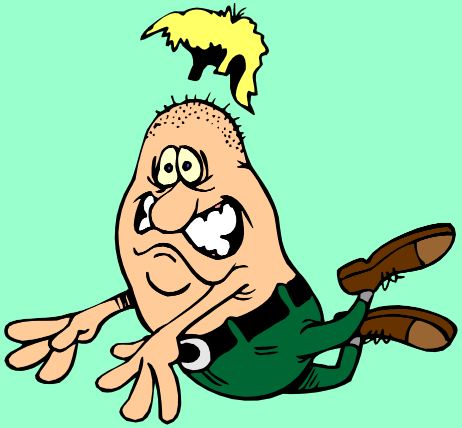



# 3. Derivative of $(ax+b)^n$

$$\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1}$$



$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$



## **Derivative of a composite function**

We have already met composite functions or “a function of a function” in Unit 1. We are now going to find an efficient method of differentiating this type of function.

Lets start with: $(3x+1)^2 = (3x+1)(3x+1)$

$$= 3x(3x+1) + 1(3x+1)$$

$$= 9x^2 + 3x + 3x + 1$$

$$= 9x^2 + 6x + 1$$

$$\Rightarrow \frac{dy}{dx} = 18x + 6$$

$$= 6(3x+1)$$

$$= 3 \times 2(3x+1)^{2-1}$$

This process takes a lot of working .
Can we relate the result to the original function?

# The Chain Rule

We know from our work on composite functions that $(3x+1)^2$

is composed of an **OUTER** function and an **INNER** function.

If we let ***u*** represent the inner function then:

OUTER FUNCTION: $u^2$

INNER FUNCTION: $u = 3x + 1$

Then derivative is: $2u$

Then derivative is: $3$

$= 2(3x + 1)$

We have shown previously that:

$$\frac{dy}{dx} = 18x + 6 \qquad = 3 \times 2(3x+1)^{2-1}$$

This process is known as **the chain rule** and can be written as follows:

# The Chain Rule

This process is known as **the chain rule** and can be written as follows:

$$\frac{d}{dx}\left(ax+b\right)^{n} = an(ax+b)^{n-1}$$

OR

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

It is more easily remembered as:

$$OUTER' \times INNER'$$

Example 1

$$OUTER' \times INNER'$$

Differentiate:

$(a)\ \ y = (5x-2)^3$

$(b)\ \ y = (x+3)^{-4}$

**NAB**

$(c)\ \ y = \dfrac{1}{3x-3}$

Solution:

$(a)\ \ y = (5x-2)^3$

$$\frac{dy}{dx} = 3 \times (5x-2)^{3-1} \times 5$$

$$\frac{dy}{dx} = 15(5x-2)^2$$

$(b)\ \ y = (x+3)^{-4}$

$$\frac{dy}{dx} = -4(x+3)^{-4-1} \times 1$$

$$\frac{dy}{dx} = -4(x+3)^{-5}$$

$(c)\ \ y = \left(3x-3\right)^{-1}$

$$\frac{dy}{dx} = -1(3x-3)^{-1-1} \times 3$$

$$\frac{dy}{dx} = -3(3x-3)^{-2}$$

Heinemann, p.269, EX 14G, Q1(a) to (c)
(f) to (h)

This is not the end

## The Chain Rule with trig functions

$$OUTER' \times INNER'$$

The chain rule can be applied to trig functions in the same way.

Example 2

Differentiate:

$(a)\ y = \cos(5x - 2)$

$$\frac{dy}{dx} = -\sin(5x-2) \times 5$$

$$\frac{dy}{dx} = -5\sin(5x-2)$$

$(b)\ y = \sin(3x^2)$

$$\frac{dy}{dx} = \cos(3x^2) \times 6x$$

$$\frac{dy}{dx} = 6x\cos(3x^2)$$

$(c)\ y = \frac{dx}{\sin^2 x}$

$$y = (\sin x)^{-2}$$

$$\frac{dy}{dx} = -2 \times (\sin x)^{-3} \times \cos x$$

$$\frac{dy}{dx} = -\frac{2\cos x}{\sin^3 x}$$

Heinemann, p.271, EX 14H, Q2 & 3