



3.

Definite Integrals

Integrals with limits



Integrals With Limits

Suppose that $F'(x) = f(x)$ so that $\int f(x) dx = F(x)$

Definition

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a) \quad (\text{must learn!!})$$

a – lower limit and b – upper limit

so

$$\begin{aligned} \int_2^4 2x dx &= [x^2 + C]_2^4 \\ &= (16 + C) - (4 + C) \\ &= 16 + C - 4 - C \\ &= \underline{12} \end{aligned}$$

NB: when using limits the Cs just cancel each other out so can be disregarded !!

Examples

Evaluate :

$$(a) \int_0^3 4x^2 dx$$

$$= \left[\frac{4x^3}{3} \right]_0^3$$

$$= \left[\frac{4}{3} \times (3)^3 \right] - \left[\frac{4}{3} \times (0)^3 \right]$$

$$= 36 - 0$$

$$= 36$$

$$(b) \int_2^4 \frac{dx}{x^3} = \int_2^4 x^{-3} dx$$

$$= \left[\frac{x^{-2}}{-2} \right]_2^4 = \left[-\frac{1}{2x^2} \right]_2^4$$

$$= \left[-\frac{1}{2(4^2)} \right] - \left[-\frac{1}{2(2^2)} \right]$$

$$= -\frac{1}{32} + \frac{1}{8}$$

$$= \frac{3}{32}$$

Examples

Evaluate :

$$(c) \int_{-2}^1 (3t-1)^2 dx \quad \begin{array}{l} (3t-1)^2 \\ = (3t-1)(3t-1) \end{array}$$

$$= \int_{-2}^1 (9t^2 - 6t + 1) dx$$

$$= \left[\frac{9t^3}{3} - \frac{6t^2}{2} + t \right]_{-2}^1$$

$$= \left[3t^3 - 3t^2 + t \right]_{-2}^1$$

$$= (3 - 3 + 1) - (-24 - 12 - 2)$$

$$= 1 - (-38) = 39$$

Examples

Evaluate :

$$(d) \int_1^{16} \left(\frac{1-2x}{2\sqrt{x}} \right) dx$$

$$= \int_1^{16} \left(\frac{1}{2\sqrt{x}} - \frac{2x}{2\sqrt{x}} \right) dx$$

$$= \int_1^{16} \left(\frac{1}{2} x^{-\frac{1}{2}} - x^{\frac{1}{2}} \right) dx$$

$$= \left[\frac{1}{2} \times \frac{2}{1} \times x^{\frac{1}{2}} - \left(\frac{2}{3} \times x^{\frac{3}{2}} \right) \right]_1^{16}$$

$$= \left[x^{\frac{1}{2}} - \left(\frac{2x^{\frac{3}{2}}}{3} \right) \right]_1^{16}$$

$$= \left[\sqrt{x} - \left(\frac{2\sqrt{(x^3)}}{3} \right) \right]_1^{16}$$

$$= \left[\sqrt{16} - \left(\frac{2\sqrt{(16^3)}}{3} \right) \right]$$

$$- \left[\sqrt{1} - \left(\frac{2\sqrt{(1^3)}}{3} \right) \right]$$

Examples

Evaluate :

$$(d) = \left[\sqrt{16} - \left(\frac{2\sqrt{(16^3)}}{3} \right) \right] - \left[\sqrt{1} - \left(\frac{2\sqrt{(1^3)}}{3} \right) \right]$$

$$= \left(4 - 42\frac{2}{3} \right) - \left(1 - \frac{2}{3} \right)$$

$$= -38\frac{2}{3} - \frac{1}{3}$$

$$= -39$$

Heinemann, p.169, EX 9L,
Q1
Q2 (a) to (h)