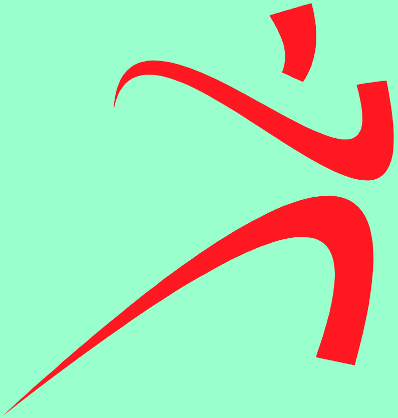


3. Composition of Functions

COMPOSITE FUNCTION-

Composed of two or more simple functions

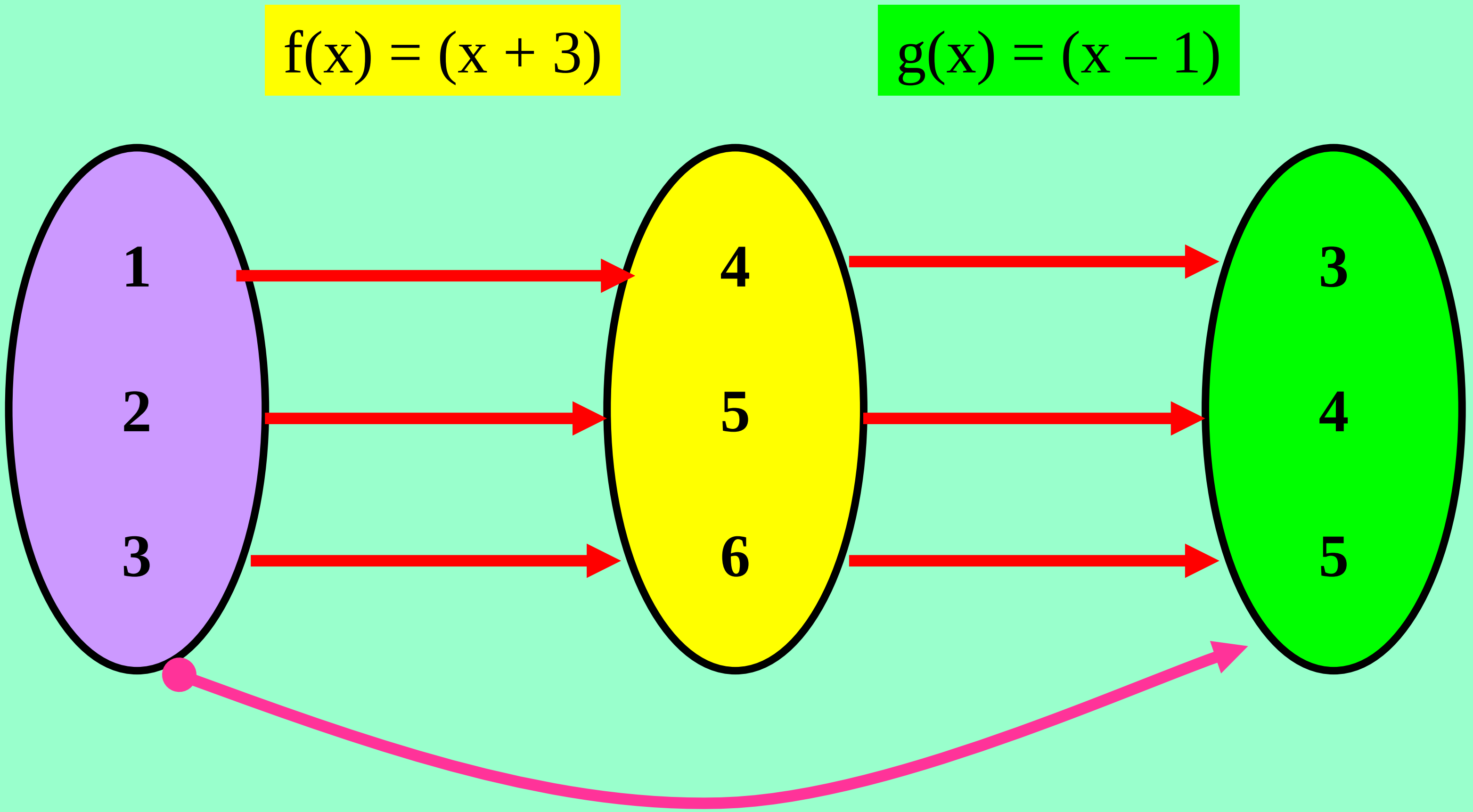
Decomposing Functions

Many functions can be thought of as being made up of other functions. For instance:

	Function 1	Function 2
$f(x) = 2x + 4$	x times 2	x add 4
$f(x) = 4x^3$	x cubed	x times 4
$f(x) = (x - 5)^2$	x - 5	x^2
$f(x) = 7(x + 6)$	x add 6	x times 7

Composition of Functions

Arrows can be used to illustrate the process of composing a function:



h(x) = x + 2 = ((x + 3) - 1) = g applied to f

Composite Functions

Copy the following

Functions which are the result of one function being applied to the result of another are called **Composite Functions**.

g applied to f is written as **g(f(x))** — "g of f of x"

f applied to g is written as **f(g(x))** — "f of g of x"

NOTE : In general $f(g(x)) \neq g(f(x))$

Example 1

If $f(x) = 3x - 1$ and $g(x) = 2x + 3$ find:

(a) $f(g(3))$ (b) $g(f(x))$

Solution:

(a)

1. If given a number work out value of inner function for that number.

2. Put this value into the outer function.

$g(x) = 2x + 3$

$g(\mathbf{3}) = 2(\mathbf{3}) + 3$

$g(3) = 6 + 3$

$g(3) = 9$

$f(x) = 3x - 1$

$f(\mathbf{9}) = 3(\mathbf{9}) - 1$

$f(9) = 26$

Example 1

If f(x) = 3x – 1 and g(x) = 2x + 3 find:

(a) f(g(3)) (b) g(f(x))

Solution:

(b)

1. If not given a number write down outer function.

2. Wherever you see x replace it with inner function.

3. Tidy up and simplify

Wherever you see an x in the outer function replace it with the inner function.

g(x) = 2x + 3

g(**f**) = 2(**f**) + 3

g(f(x)) = 2(3x – 1) + 3

g(f(x)) = 6x – 2 + 3

g(f(x)) = 6x + 1

Example 2

If $f(x) = x^2 + 1$ and $g(x) = \dfrac{1}{x^2}$ find:

(a) $h(x) = f(g(x))$ (b) $k(x) = g(f(x))$

Solution to (a):

1. If not given a number write down outer function.

$f(x) = x^2 + 1$

2. Wherever you see x replace it with inner function.

$f(\mathbf{g}) = (\mathbf{g})^2 + 1$

$$f(g(x)) = \left(\frac{1}{x^2}\right)^2 + 1$$

3. Note that a suitable domain cannot include 0 as we would have to divide by zero

$$h(x) = \frac{1}{x^4} + 1 \quad , x \neq 0$$

Example 2

If $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x^2}$ find:

(a) $h(x) = f(g(x))$ (b) $k(x) = g(f(x))$

Solution to (b):

1. If not given a number write down outer function.

2. Wherever you see x replace it with inner function.

NOTE ;
f(g(x)) does not equal g(f(x))

$$g(x) = \frac{1}{x^2}$$

$$g(\mathbf{f}) = 1 / (\mathbf{f})^2$$

$$g(f(x)) = \frac{1}{(x^2+1)^2} = \frac{1}{(x^2+1)(x^2+1)}$$

$$k(x) = \frac{1}{x^4+2x^2+1}$$

Heinemann , p.26, EX 2C

Q3, 4, 5(a) to (e)