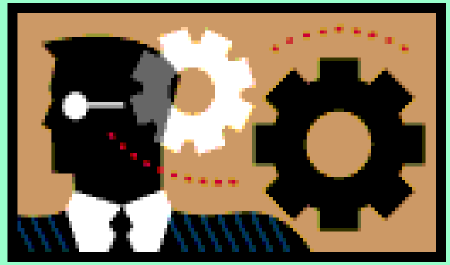
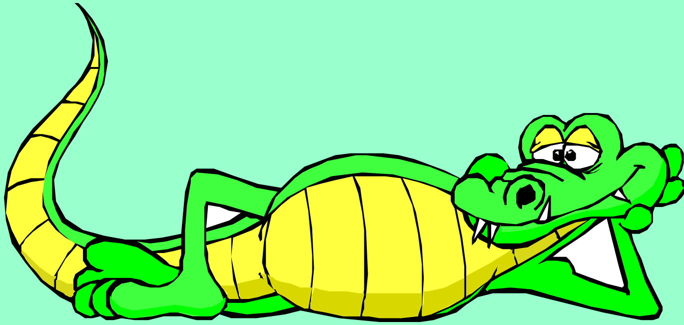
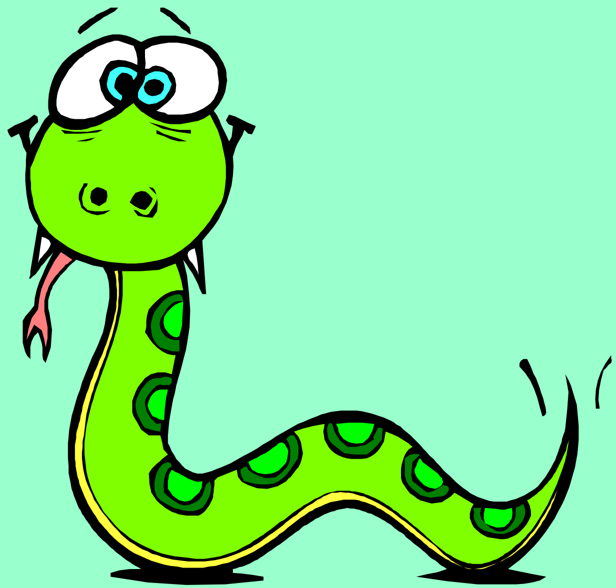




3.

Derivatives of $f(x) = ax^n$



The Derivative

Copy the following:

The process of deriving $f'(x)$ from f(x) is called **differentiation.**

$f'(x)$ represents two things:

- The rate of change of the function
- The gradient of the tangent to the function

Power must be rational

Basic Rule

If $f(x) = ax^n$ then $f'(x) = nax^{n-1}$

multiply by the power → **reduce the power by 1**

Example 1

Find the derivative of:

(a) x^9

$= 9x^{(9-1)}$

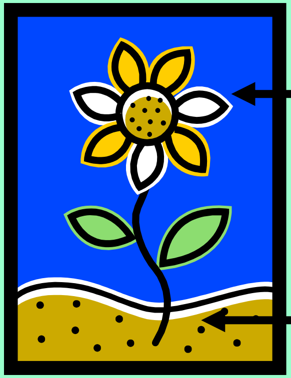
$= 9x^8$

(b) $4x^{-9}$

$= -9 \times 4x^{(-9-1)}$

$= -36x^{-10}$

Think "Flower Power":



Power on top

Root at bottom

(c) $\sqrt[5]{x^3}$

$= x^{\frac{3}{5}}$

Must be if form ax^n

$= \frac{3}{5} x^{(\frac{3}{5}-\frac{5}{5})}$

$= \frac{3}{5} x^{-\frac{2}{5}}$

(d) $\frac{2}{\sqrt{x^7}}$

$= 2x^{-\frac{7}{2}}$

$= -\frac{7}{2} \times 2x^{(-\frac{7}{2}-\frac{2}{2})}$

$= -7x^{-\frac{9}{2}}$

Heinemann , p.95, EX 6F, Q1-10

Example 2

Find the gradient of the tangent to the curve $f(x) = \frac{1}{x^5}$ at x = 2

Solution:

1. Prepare for differentiation (ie must be if form **ax^n**)

2. **"Multiply by power then reduce power by 1"**

3. Tidy up

4. Substitute in given value for x

$$f(x) = \frac{1}{x^5}$$

$$f(x) = x^{-5}$$

$$f'(x) = -5 \times 1x^{(-5-1)}$$

$$f'(x) = -5x^{-6}$$

$$f'(x) = \frac{-5}{x^6}$$

$$f'(2) = \frac{-5}{(2)^6}$$

$$f'(2) = -\frac{5}{64}$$

Example 3

The volume in a container can be calculated using $V(t) = \sqrt[3]{t^2}$

Calculate the rate of change of the volume after 8 seconds.

Solution:

1. Prepare for differentiation (ie must be if form **ax^n)**

2. **"Multiply by power then reduce power by 1"**

3. Tidy up

4. Substitute in given value for t

$$V(t) = \sqrt[3]{t^2}$$

$$V(t) = t^{\frac{2}{3}}$$

$$V'(t) = \tfrac{2}{3} t^{\left(\frac{2}{3} - \frac{3}{3}\right)}$$

$$V'(t) = \tfrac{2}{3} t^{-\frac{1}{3}}$$

$$V'(t) = \frac{2}{3} \times \frac{1}{\sqrt[3]{t}}$$

$$V'(8) = \frac{2}{3} \times \frac{1}{\sqrt[3]{8}}$$

Example 3

The volume in a container can be calculated using $V(t)=\sqrt[3]{t^2}$

Calculate the rate of change of the volume after 8 seconds.

Solution:

1. Prepare for differentiation (ie must be if form $\mathbf{ax^n}$)

2. **"Multiply by power then reduce power by 1"**

3. Tidy up

4. Substitute in given value for t

$$V'(8)=\frac{2}{3}\times\frac{1}{\sqrt[3]{8}}$$

$$V'(8)=\frac{2}{3}\times\frac{1}{2}$$

$$V'(8)=\frac{2}{6}=\frac{1}{3}$$

Heinemann , p.92, EX 6E