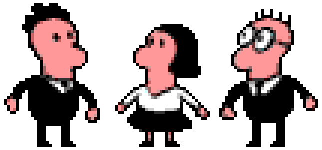
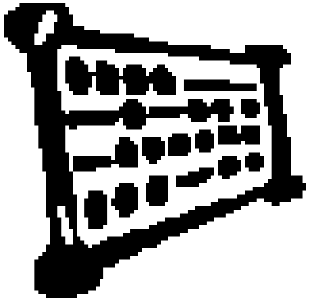
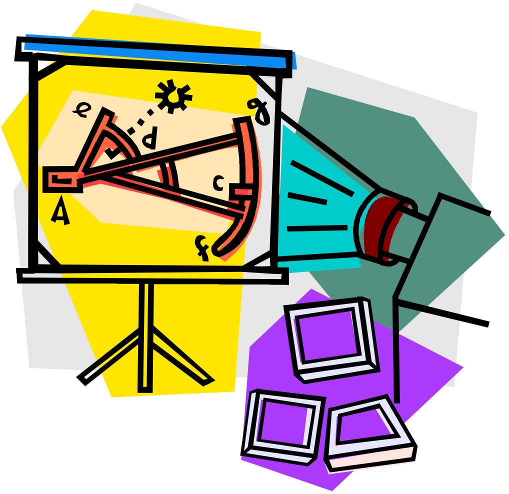


2. The Derived Function





Required skills

Before we start ……….

You will need to remember work with Indices, as well as what you have learned about Straight Lines from Unit 1.1.

Lets recall the rules on indices …..

Rules of indices

Rule	*Examples*
$a^0 = 1$	$12.314^0 = 1$
$a^{-m} = \frac{1}{a^m}$	$x^{-5} = \frac{1}{x^5}$ $\quad \frac{1}{n^3} = n^{-3}$
$\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$\sqrt[3]{x^2} = x^{\frac{2}{3}}$ $\quad y^{\frac{6}{5}} = \sqrt[5]{y^6}$
$a^m \times a^n = a^{m+n}$	$2a^{3/2} \times 3a^{1/2} = 6a^{\frac{3}{2}}a^{\frac{1}{2}}$ $= 6a^2$
$a^m \div a^n = a^{m-n}$	$x^2 \div x^{-3} = x^{2--3}$ $= x^5$
$(a^m)^n = a^{mn}$	$(q^2)^3 = q^2 \times q^2 \times q^2$ $= q^6$

What is Differentiation?

Differentiation is the process of deriving

$f'(x)$ from $f(x)$. We will look at this process in a second.

$f'(x)$ is called the derived function or derivative of $f(x)$.

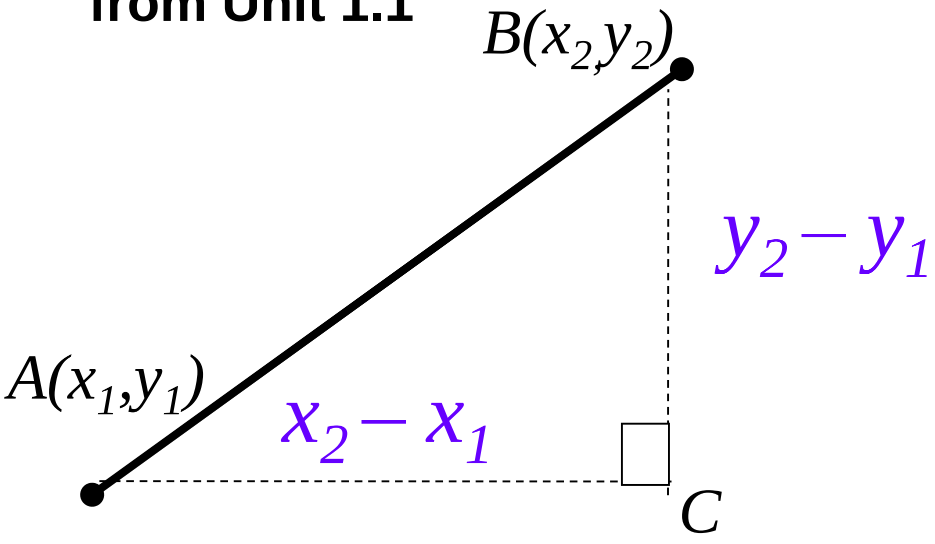
The derived function represents:

- the rate of change of the function
- the gradient of the tangent to the graph of the function.

Tangents to curves

The derivative function is a measure of the gradient or slope of a function at any given point. This requires us to consider the gradient of a line.

We can do this if we think about how we measure the gradient from Unit 1.1



The gradient of AB = m_{AB}

$$= \frac{\text{Change in y}}{\text{Change in x}}$$

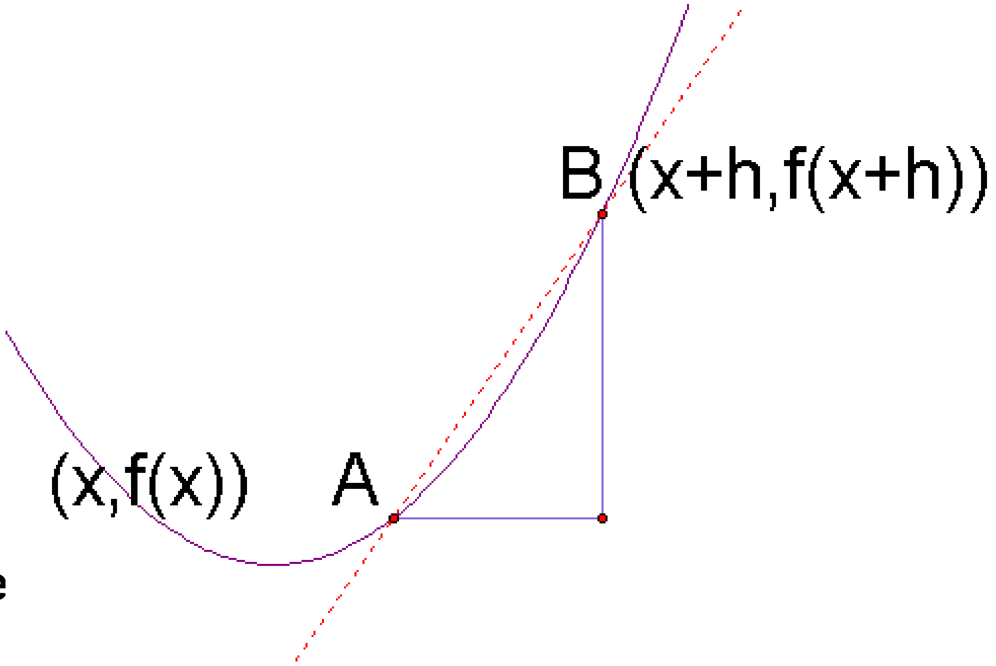
$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad \text{Gradient Formula}$$

Tangents to curves

We will look at a function and think about the gradient of the function at any given point. The function itself is not important, the process we go through to get the gradient is. We want to find the gradient of a curve.

A is the point (x, f(x)) and B is a point on the function a short distance h from A. This gives B the coordinates (x+h, f(x+h))

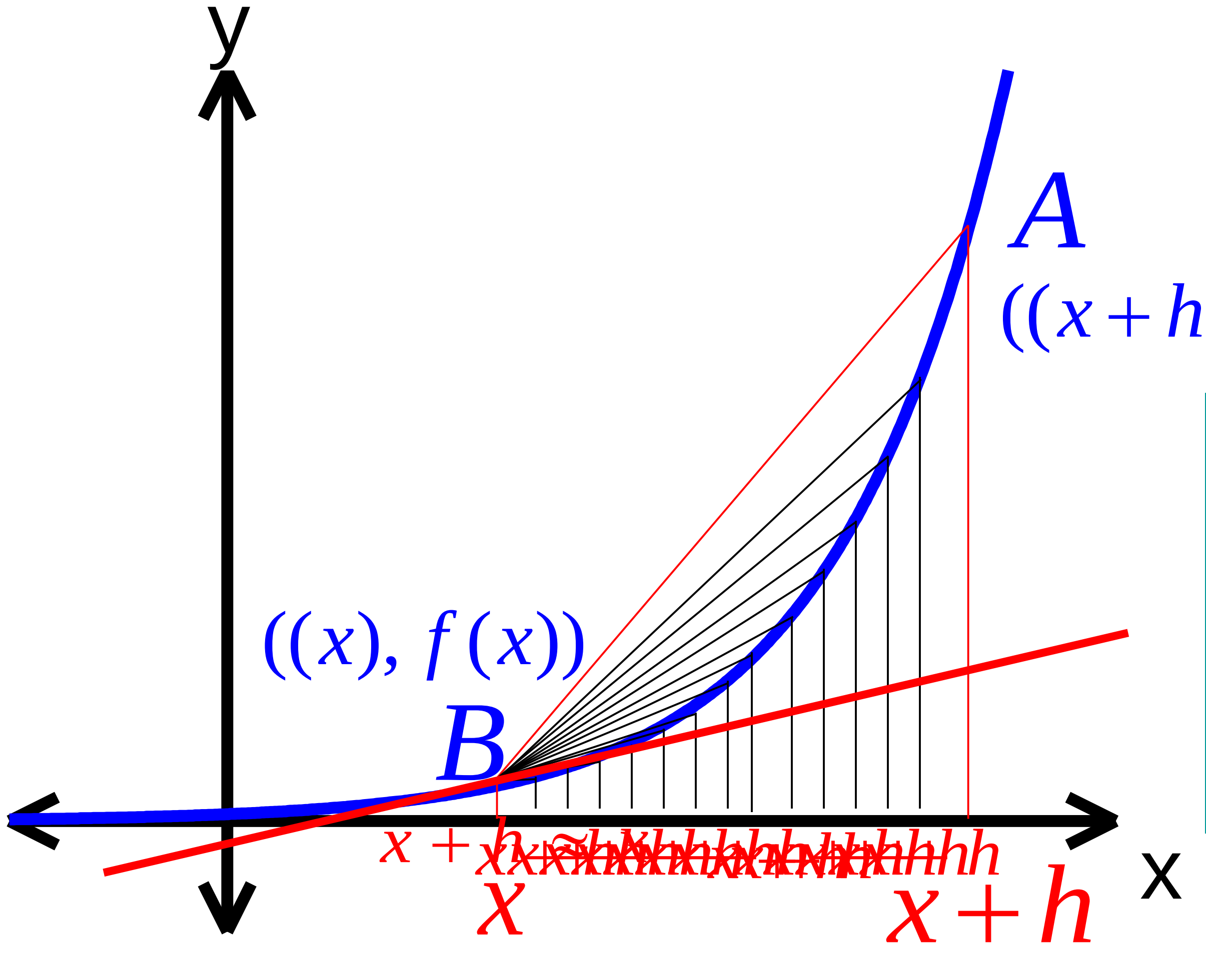


The line AB is shown on the diagram. We want to find the gradient of the curve at A. If we find the gradient of the line AB and move B towards A we should get the gradient at A.

What happens if we make h smaller?

$$M_{AB} = \frac{f(x+h) - f(x)}{(x+h) - x}$$

$y = f(x)$

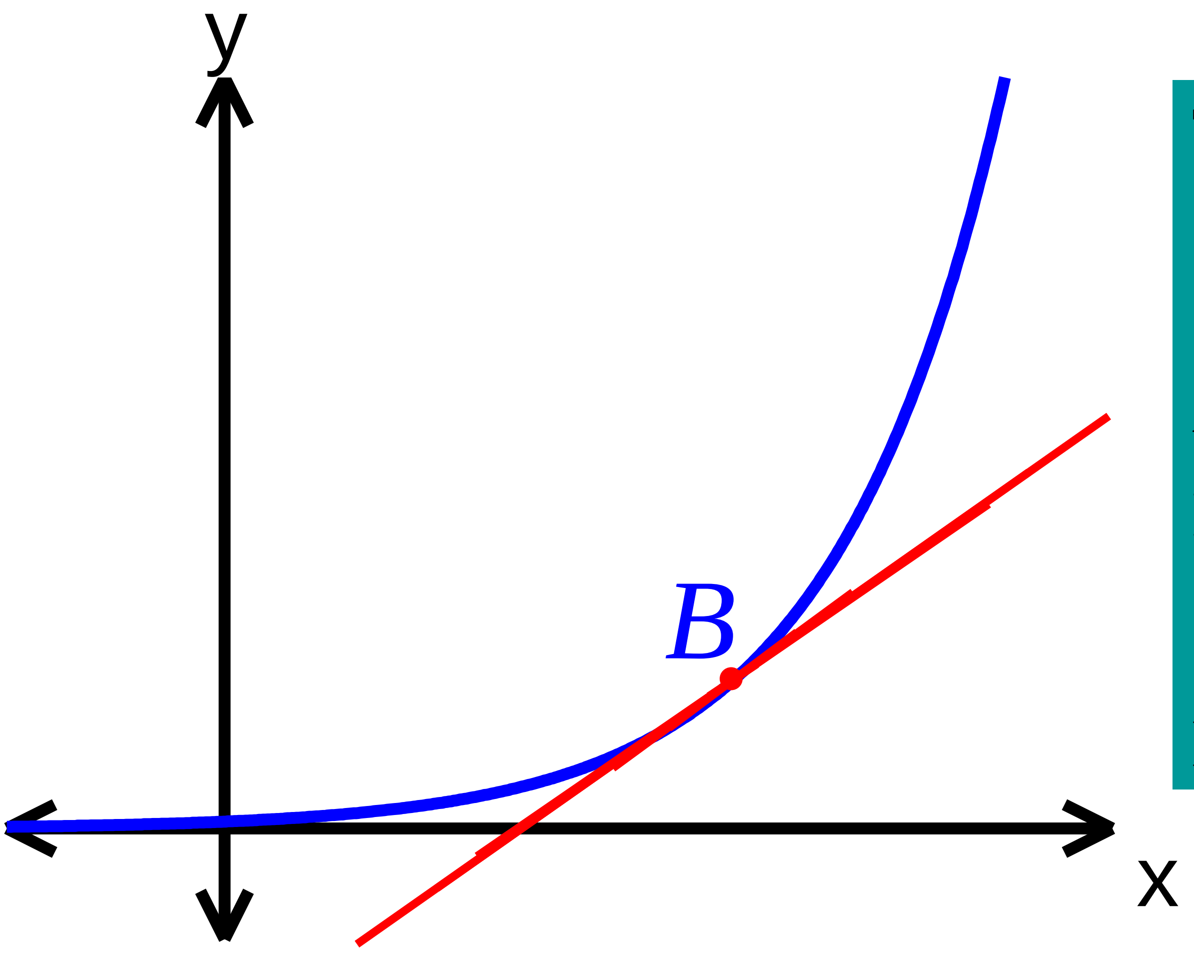


When $x + h \approx x$

Then gradient of AB is the gradient of the tangent at B

Shrinking the tangent line is the same as letting h get very small

$y = f(x)$



The gradient doesn't change as the line gets smaller and so the gradient at B must be the same as the gradient of the line

What are we saying?

We know

$$M_{AB} = \frac{f(x+h) - f(x)}{(x+h) - x}$$

But

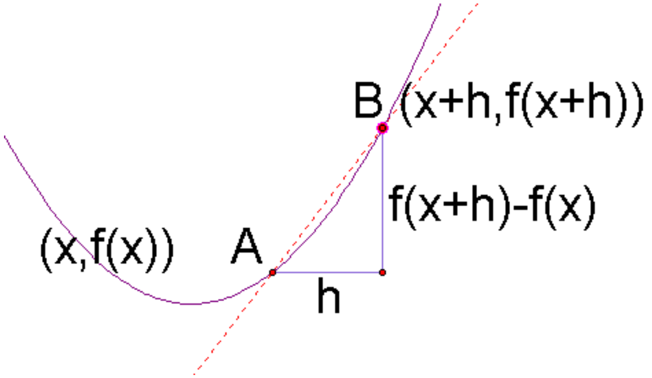
$$M_{B} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$$

This is the basic, first principle definition of the derived function. Usually written $f^{/}(x)$

Putting it together….

Differentiate the function f(x) = x^2 *from first principles.*

The derivative is the same as the gradient of the tangent to the curve so we can go straight to the gradient formula we saw in the previous slides.



The limit as h → 0 is written as $\lim_{h \to 0}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h(2x+h)}{h}$$

$$f'(x) = \lim_{h \to 0} (2x+h)$$

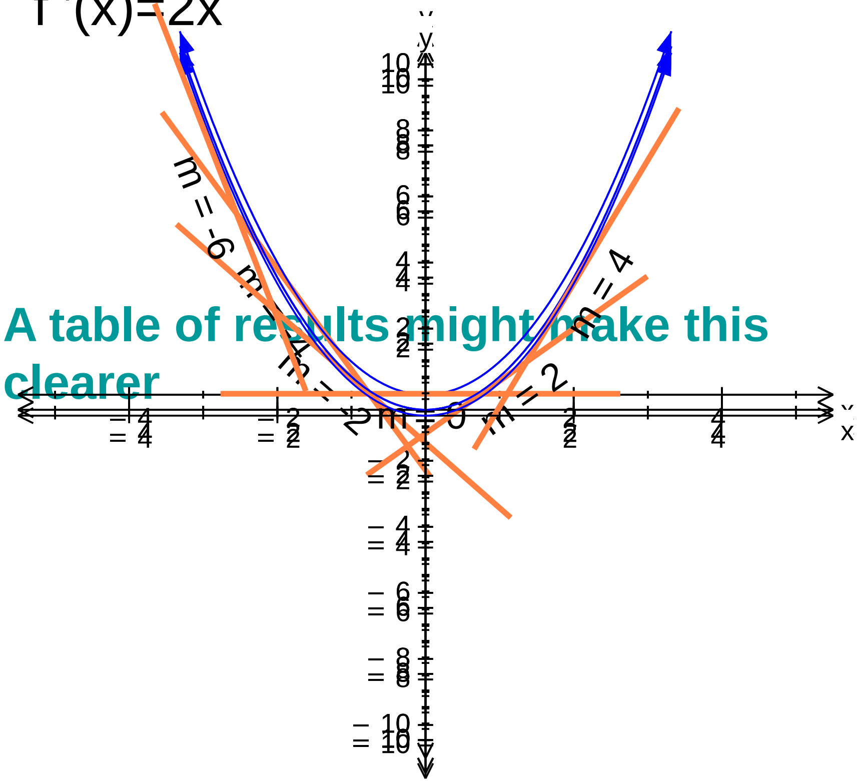
$$f'(x) = 2x$$

h gets so small its effectively zero.

What does the answer mean?

For each and every point on the curve $f(x)=x^2$, the gradient of the tangent to the curve is given by the formula

$f'(x)=2x$

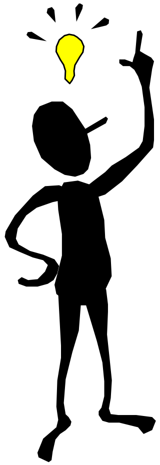


A table of results might make this clearer

Value of x	*Gradient of the tangent*
-3	*-6*
-2	*-4*
-1	*-2*
0	*0*
1	*2*
2	*4*
3	*6*
4	*8*

Is there an easier way to do this?

Rules for differentiation



There are four rules for differentiating – remember these and you can differentiate anything …

Rule	*Examples*
$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$	$f(x) = x^6 \Rightarrow f'(x) = $ **$6x^{6-1}$** = **$6x^5$**
$f(x) = cx^n \Rightarrow f'(x) = cnx^{n-1}$	$f(x) = 4x^2 \Rightarrow f'(x) = $ **$4 \times 2x^{2-1}$** = **$8x^1$ or $8x$**
$f(x) = c \Rightarrow f'(x) = 0$	$f(x) = 65 \Rightarrow f'(x) = $ **0**
$f(x) = g(x) + h(x) \Rightarrow f'(x) = g'(x) + h'(x)$	$f(x) = x^6 + 4x^2 + 65 \Rightarrow f'(x) = $ **$6x^5 + 8x$**