

2.

Logarithms



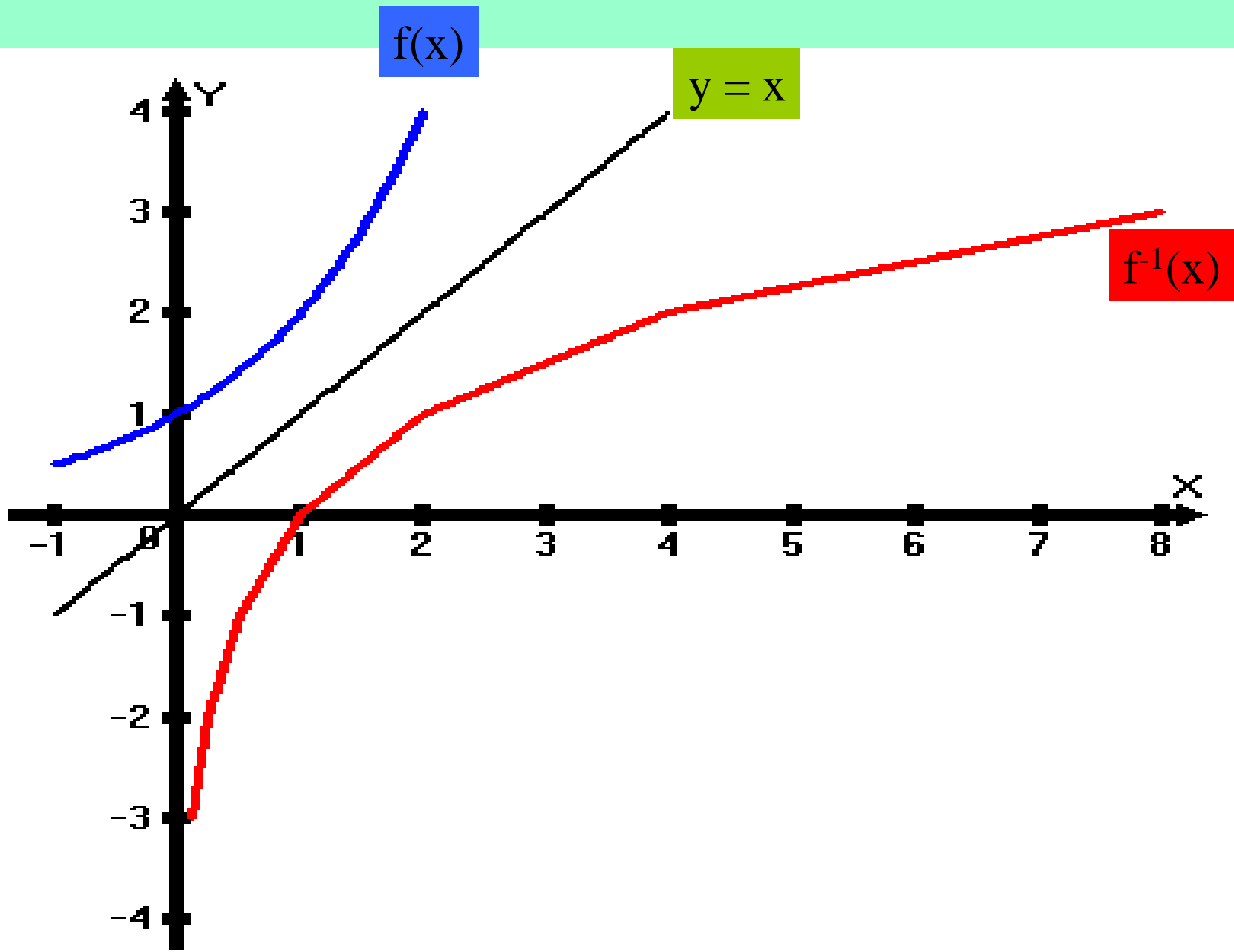
The Logarithmic Function

Recall that to find the inverse of a function we reflect that function in the line $y = x$.

In essence this means **swapping the x and y co-ordinates**. For $f(x) = 2^x$ this leads to:

x	$1/8$	$1/4$	$1/2$	1	2	4	8
$f^{-1}(x)$	-3	-2	-1	0	1	2	3

Plotting these points and $f(x) = 2^x$ on the same graph along with the line $y = x$ we can see they are inverses:



The Logarithmic Function

The inverse of the exponential function is the logarithmic function.

This has the form:

$$y = \log_a x$$

“log base a of x”

x must be positive

When dealing with log functions we must think:

“the base to what power gives x” or $a^y = x$

Example 1

$$y = \log_a x \longleftrightarrow a^y = x$$

Write in logarithmic form:

(a) $3^4 = 81$

(b) $p = q^5$

(c) $16^{1/2} = 4$

Solution:

(a) $3^4 = 81$

(b) $p = q^5$

(c) $16^{1/2} = 4$

$$4 = \log_3 81$$

$$\frac{1}{2} = \log_{16} 4$$

$$\log_3 81 = 4$$

$$\log_q p = 5$$

$$\log_{16} 4 = \frac{1}{2}$$

Heinemann, p.286, EX 15E, Q1(a) to (i)

This is not the end

Example 2

$$y = \log_a x \longleftrightarrow a^y = x$$

Simplify :

NAB

(a) $\log_2 8$

(b) $\log_4 256$

(c) $\log_6 6$

Solution:

$2^x = 8$

$4^x = 256$

$6^x = 6$

(a) $\log_2 8$

(b) $\log_4 256$

(c) $\log_6 6$

= 3

= 4

= 1

An exact
solution for

$a^x = n$ is:

$$x = \frac{\log(n)}{\log(a)}$$

NB : $\log_a a = 1$

Heinemann, p.286, EX 15E, Q2(a) to (e)

This is not the end

Example 3

$$y = \log_a x \longleftrightarrow a^y = x$$

Change to exponential form:

$$(a) \ x = \log_y 7 \qquad (b) \ 3b = \log_c 8 \qquad (c) \ 4p + 2 = \log_q 12$$

Solution:

$$(a) \ x = \log_y 7 \qquad (b) \ 3b = \log_c 8 \qquad (c) \ 4p + 2 = \log_q 12$$

$$\Rightarrow y^x = 7$$

$$\Rightarrow c^{3b} = 8$$

$$\Rightarrow q^{(4p+2)} = 12$$

Heinemann, p.286, EX 15E, Q3

This is not the end

Example 4

NAB

$$y = \log_a x \longleftrightarrow a^y = x$$

If $\log_5 y = 2 \cdot 8$ write down an expression for the exact value of y .

Solution:

$$\log_5 y = 2 \cdot 8$$

$$\Rightarrow y = 5^{2 \cdot 8}$$