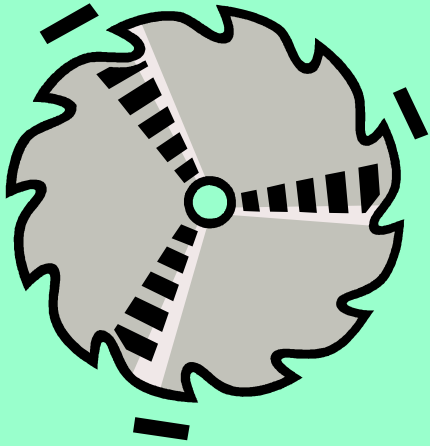


2.

## Equation of Circle - Centre (a,b)

$$(x - a)^2 + (y - b)^2 = r^2$$



## Equation of a circle with centre (a,b)

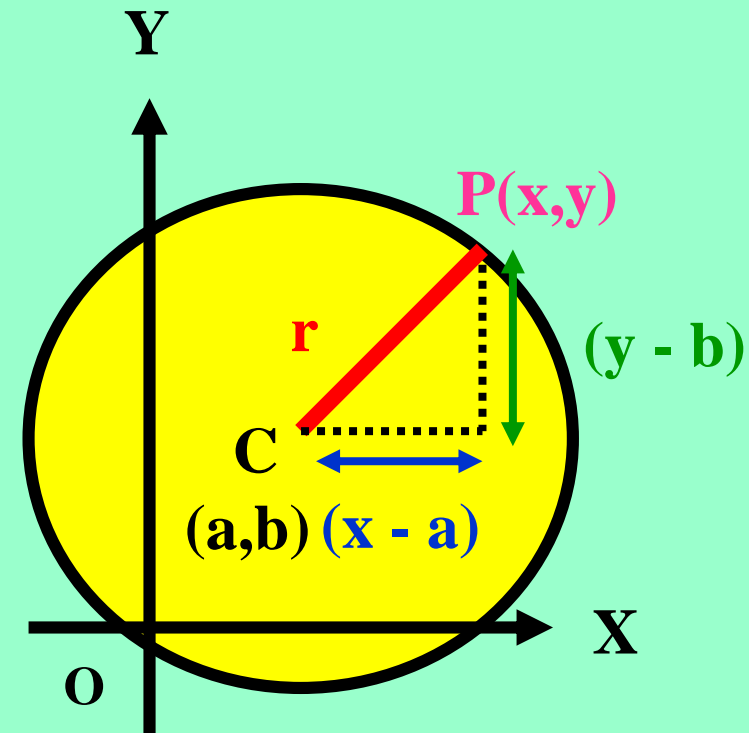
Take any point on the circumference of a circle which has the point C (a,b) as its centre.

Applying the distance formula we get:

$$r^2 = (x - a)^2 + (y - b)^2$$

This gives us the equation for a circle with centre, C (a,b) :

$$(x - a)^2 + (y - b)^2 = r^2$$



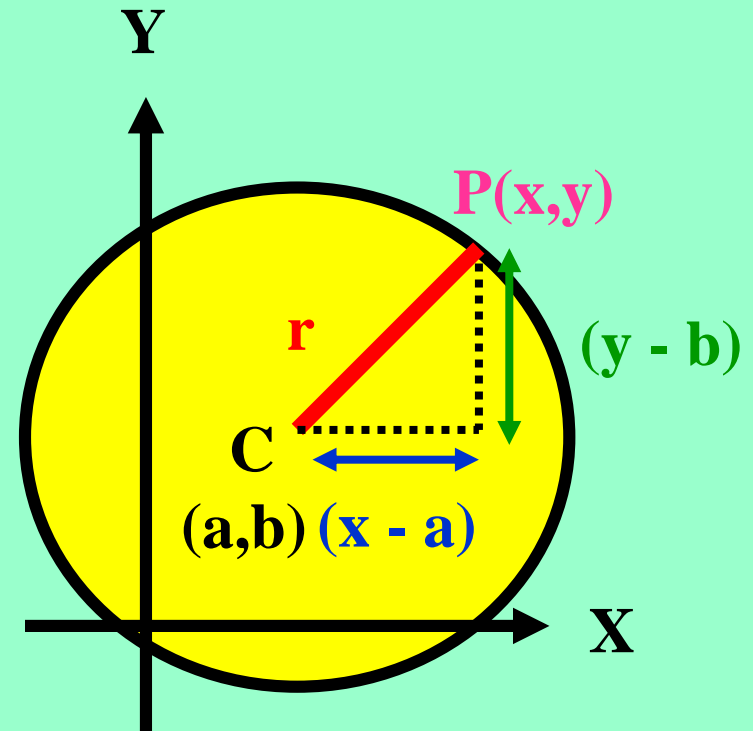
## Equation of a circle with centre (a,b)

Copy the following:

The equation of any circle with centre C (a,b) and radius r is:

$$(x - a)^2 + (y - b)^2 = r^2$$

CHANGE SIGN



If a point (x,y) lies:

**INSIDE** the circle then  $(x - a)^2 + (y - b)^2 < r^2$

**OUTSIDE** the circle then  $(x - a)^2 + (y - b)^2 > r^2$

**ON THE CIRCUMFERENCE** then  $(x - a)^2 + (y - b)^2 = r^2$

## Example 1

(a) These circles have centre (3, -4). Write down their equations:

(i) if  $r = 3$

$$(x-3)^2 + (y+4)^2 = 3^2$$

$$(x-3)^2 + (y+4)^2 = 9$$

(ii) if the circle passes through (-1,-8)

For all points :  $(x - a)^2 + (y - b)^2 = r^2$

Have a and b  
so need  $r^2$

$$r^2 = (-1-3)^2 + (-8+4)^2$$

$$r^2 = 16 + 16$$

$$r^2 = 32$$

Equation for  
all points:

$$(x-3)^2 + (y+4)^2 = 32$$

## Example 1

(b) Find the centre and radius of the circle with equation:

(i)  $(x-4)^2 + (y-1)^2 = 9$

Centre (4 , 1)

$$r^2 = 9$$

$$r = \sqrt{9}$$

$$r = 3$$

(ii)  $(x+5)^2 + (y+7)^2 = 8$

Centre (-5 , -7)

$$r^2 = 8$$

$$r = \sqrt{8}$$

$$r = \sqrt{4 \times 2}$$

$$r = 2\sqrt{2}$$

(iii)  $(x+4)^2 + y^2 = 23$

Centre (-4 , 0)

$$r^2 = 23$$

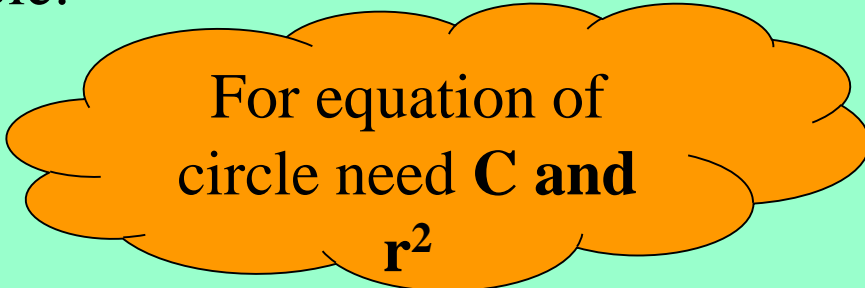
$$r = \sqrt{23}$$

## Example 2

NAB

A circle has radius 6 units and centre  $(-2, -1)$ . Write down the equation of the circle.

Solution:



For equation of circle need **C** and  **$r^2$**

$$(x - a)^2 + (y - b)^2 = r^2$$

$$(x + 2)^2 + (y + 1)^2 = 6^2$$

← Change signs

$$(x + 2)^2 + (y + 1)^2 = 36$$

### Example 3

Find the equation of the circle with a diameter which has end points (3, 5) and (9, 7)

Solution:

For equation of circle need **C** and  **$r^2$**

If we have diameter, Centre is midpoint

$$C = \left( \frac{3+9}{2}, \frac{5+7}{2} \right)$$

C is (6, 6)

Radius = distance from C to **any** point on circumference.

$$r^2 = (9-6)^2 + (7-6)^2$$

$$r^2 = 3^2 + 1^2$$

$$r^2 = 10$$

### Example 3

Find the equation of the circle with a diameter which has end points (3 , 5) and (9 , 7)

Solution:

C is (6, 6)

$$r^2 = 10$$

Equation of circle:  $(x - 6)^2 + (y - 6)^2 = 10$



## Example 4

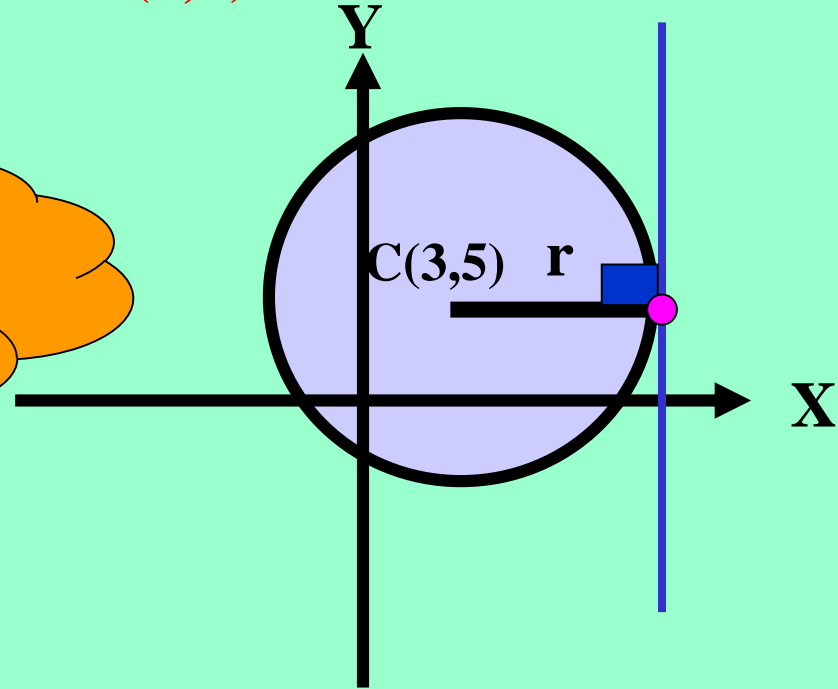
Find the equation of the circle with **centre (3,5)** and which has  $x = 7$  as a **tangent**.

**Solution:**

For equation of circle need **C** and  **$r^2$**

Centre:

**(3, 5)**



**Point of contact:**

Because tangent is vertical radius is horizontal  $\longrightarrow$  POC has same y-coord as C

So Point of Contact is **(7, 5)**

$$r^2 = (7 - 3)^2 + (5 - 5)^2 = 16$$

## Example 4

Find the equation of the circle with **centre (3,5)** and which has  $x = 7$  as a **tangent**.

Solution:

For equation of circle need **C** and  **$r^2$**

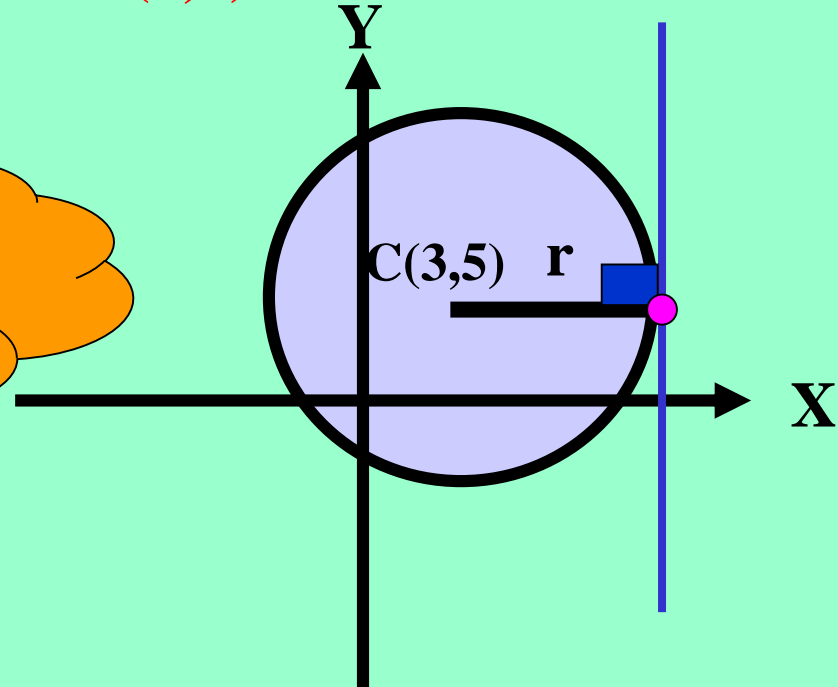
Centre:

**(3, 5)**

**$r^2 = 16$**

Equation of circle:

$$(x-3)^2 + (y-5)^2 = 16$$



Heinemann,  
p.210, EX 12F, Q1, 2, 3, 4 & 6