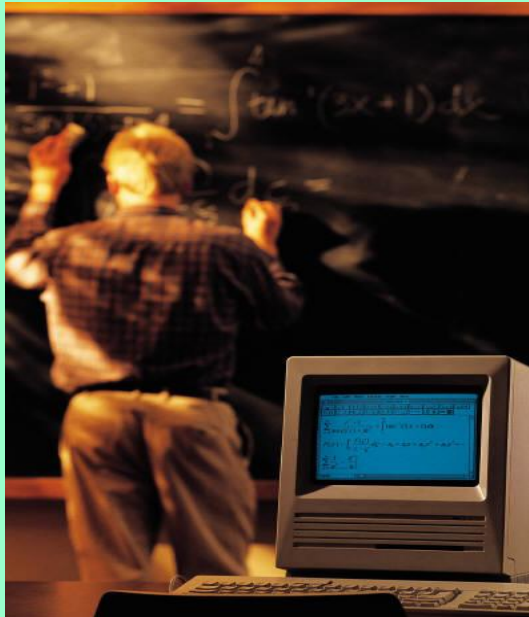
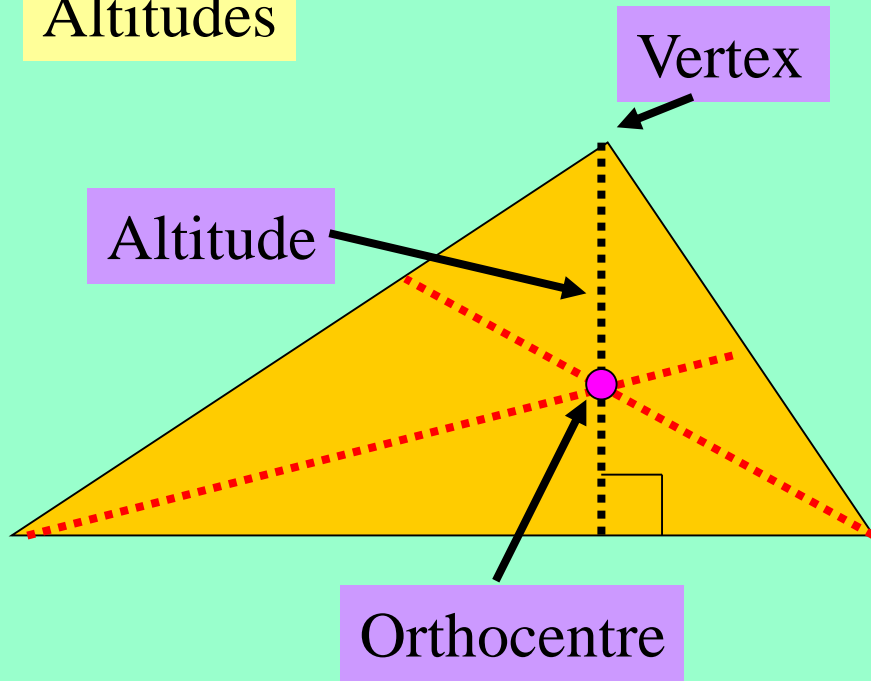


**12.**

# Triangles : Special Lines



## Altitudes



## Copy the following:

An altitude is a straight line drawn from a vertex and is **perpendicular** to the opposite side.

Every triangle has 3 altitudes.

These altitudes are **concurrent** and they intersect at the **ORTHOCENTRE**.

To find the equation of an Altitude:

1. Find the gradient of the side opposite the vertex

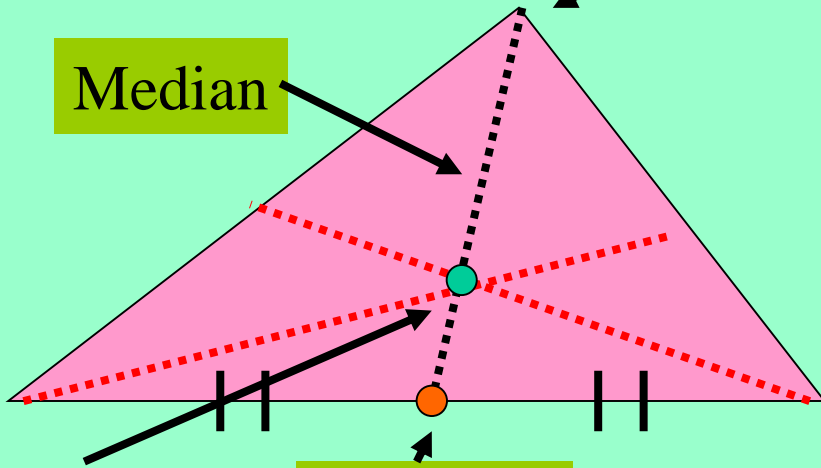
2. Find gradient of Altitude using  $m_1 \times m_2 = -1$

3. Find equation using coordinates of vertex and  $y - y_1 = m(x - x_1)$

Medians

Vertex

Median



Centroid

Midpoint

Copy the following:

A median is a line drawn from a vertex to the **Midpoint** of the side opposite the vertex.

Again there are 3 medians and they are **concurrent**.

The point of intersection is called the **CENTROID**.

To find the equation of a Median:

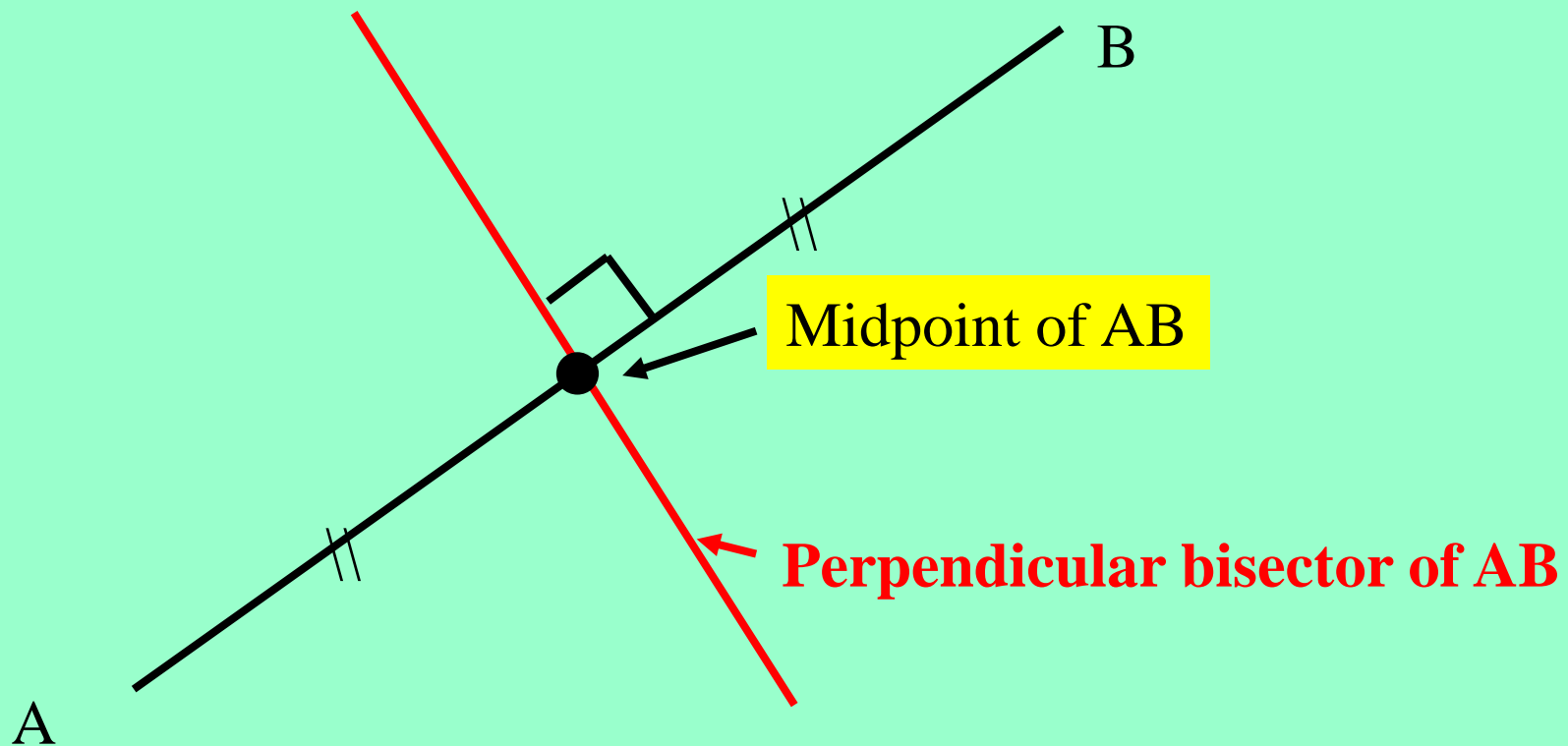
1. Find midpoint of side opposite Vertex.

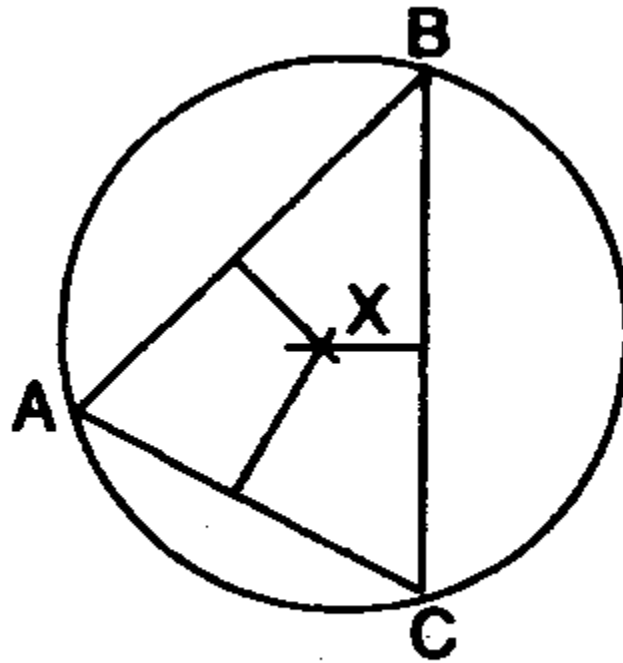
2. Find gradient between midpoint and vertex using  $m = \frac{y_2 - y_1}{x_2 - x_1}$

3. Find equation using coords of vertex and  $y - y_1 = m(x - x_1)$

# Perpendicular Bisectors

- This line does as it says -
- It cuts through the middle of another line.....
- at right angles to it.





**Fig. 63. Circumcircle. X is the circumcentre of triangle ABC.**

In triangles, the point where the perpendicular bisectors of the sides meet is called the **CIRCUMCENTRE**.

This point is also the centre of the circle that could be drawn through all three vertices of the triangle.  
(This circle is known as a circumcircle).

## To find the equation of a Perpendicular Bisector:

1. Find midpoint of given line or vertices

2. Find gradient between given points using  $m = \frac{y_2 - y_1}{x_2 - x_1}$

3. Find gradient of perpendicular bisector using  $m_1 \times m_2 = -1$

4. Find equation using coords of midpoint and  $y - y_1 = m(x - x_1)$

## Example 1

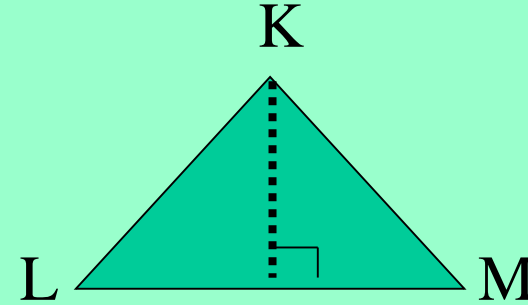
Triangle KLM has vertices K (-2 , 1), L (2 , 3) and M (4 , -3).

Find the equation of:

- the altitude drawn from K
- the median drawn from M

**Solution to (a) :**

- Draw quick sketch
- Find the gradient of the side opposite the vertex (LM)
- Find gradient of Altitude using  $m_1 \times m_2 = -1$
- Find equation using coordinates of vertex and  $y - y_1 = m (x - x_1)$



$$m_{LM} = \frac{-3 - 3}{4 - 2} = \frac{-6}{2} = -3$$

$$m_{altitude} = \frac{1}{3}$$

Flip  
fraction-  
change  
sign

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{3} (x - (-2))$$

$$y - 1 = \frac{1}{3} (x + 2)$$

**x3**

$$3y - 3 = x + 2$$

$$3y = x + 5$$

## Example 1

Triangle KLM has vertices K (-2 , 1), L (2 , 3) and M (4 , -3).

Find the equation of:

- (a) the altitude drawn from K
- (b) the median drawn from M

**Solution to (b) :**

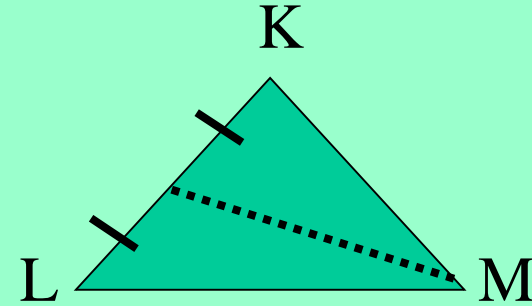
1. Draw quick sketch

2. Find midpoint of side opposite Vertex. (KL)

3. Find gradient between midpoint and vertex

4. Find equation using coordinates of vertex and

$$y - y_1 = m(x - x_1)$$



$$M \left( \frac{-2+2}{2}, \frac{1+3}{2} \right) = (0, 2)$$

$$m = \frac{-3-2}{4-0} = -\frac{5}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = -\frac{5}{4}(x - 4)$$

$$y + 3 = -\frac{5}{4}(x - 4)$$

$$4y + 12 = -5(x - 4)$$

**x4**



## Example 1

Triangle KLM has vertices K (-2 , 1), L (2 , 3) and M (4 , -3).

Find the equation of:

- (a) the altitude drawn from K
- (b) the median drawn from M

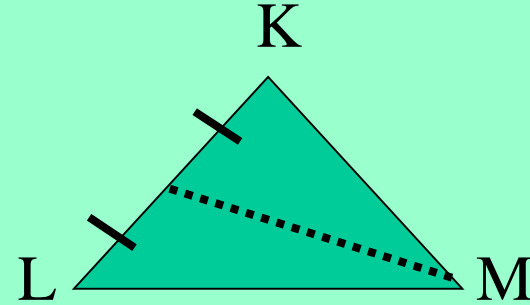
**Solution to (b) :**

1. Draw quick sketch

2. Find midpoint of side opposite Vertex. (KL)

3. Find gradient between midpoint and vertex

4. Find equation using coordinates of vertex and  $y - y_1 = m (x - x_1)$



$$4y + 12 = -5(x - 4)$$

$$4y + 12 = -5x + 20$$

$$4y + 5x = 20 - 12$$

$$4y + 5x = 8$$

## Example 2

Find the equation of the perpendicular bisector of the line joining (3, 6) and (5, 12)

**Solution :**

1. Find midpoint of given points

2. Find gradient between given points

3. Find gradient of perpendicular bisector using  $m_1 \times m_2 = -1$

4. Find equation using coords of midpoint

$$M \left( \frac{5+3}{2}, \frac{12+6}{2} \right) = (4, 9)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12-6}{5-3} = \frac{6}{2} = 3$$

$$m_{\text{perp}} = -\frac{1}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 9 = -\frac{1}{3}(x - 4) \quad \times 3$$

$$3y - 27 = -1(x - 4)$$

$$3y - 27 = -x + 4$$

$$x + 3y = 31$$

Heinemann:

p.13, Ex 1I, Q1 (Perp Bisectors)

p.15, Ex 1K, Q1 (Altitudes)

p.16, Ex 1M, Q1 (Medians)