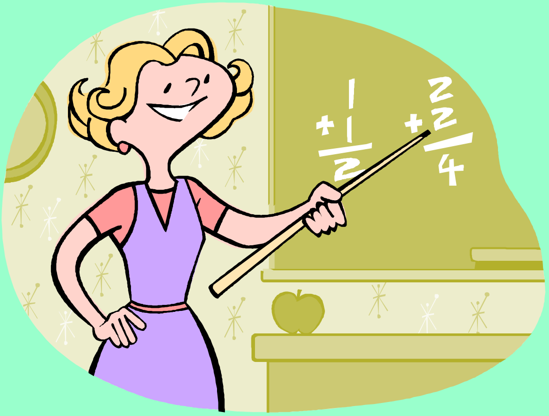
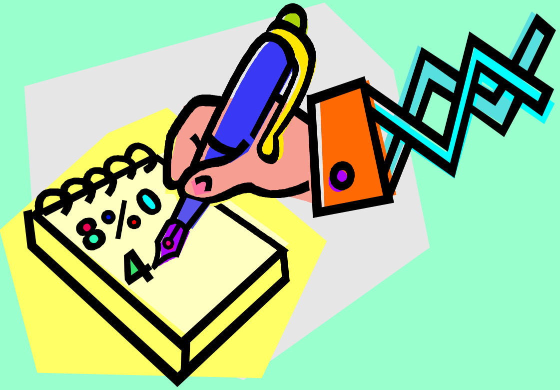
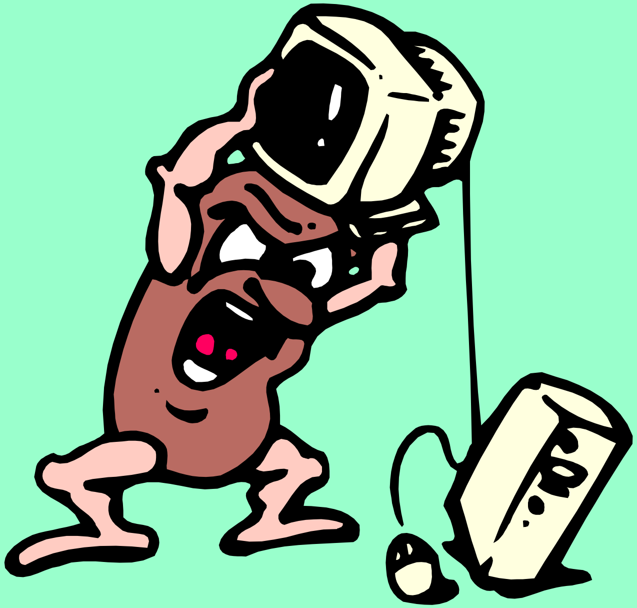




11.

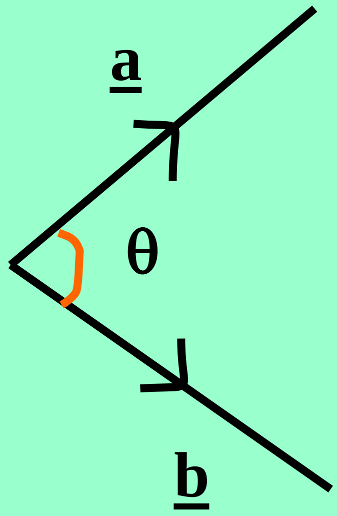
Angle between vectors



$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

Angle Between Vectors

Given that $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos\theta$



then it must follow that

$$\cos\theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}||\underline{b}|}$$

… and this allows us to find the angle between $\underline{a}$ & $\underline{b}$.

Example 1

If $\underline{m} = 2\underline{i} - \underline{j} + \underline{k}$ and $\underline{n} = \underline{i} + \underline{k}$

Find the size of the angle between $\underline{m}$ & $\underline{n}$ in radians.

Solution:

****

$\underline{m} = 2\underline{i} - \underline{j} + \underline{k}$ so $|\underline{m}| = \sqrt{(2^2 + (-1)^2 + 1^2)} = \sqrt{6}$

$\underline{n} = \underline{i} + \underline{k}$ so $|\underline{n}| = \sqrt{(1^2 + 0^2 + 1^2)} = \sqrt{2}$

$\underline{m}.\underline{n} = (2 \text{ X } 1) + (-1 \text{ X } 0) + (1 \text{ X } 1) = 2 + 0 + 1 = 3$

$$\cos\theta = \frac{\underline{m} \cdot \underline{n}}{|\underline{m}||\underline{n}|} = \frac{3}{\sqrt{6} \text{ X } \sqrt{2}} = \frac{3}{\sqrt{12}} = \frac{3}{\sqrt{4} \text{ X } \sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}({}^{\sqrt{3}}/_{2}) = 30° = {}^{\pi}/_{6}$$

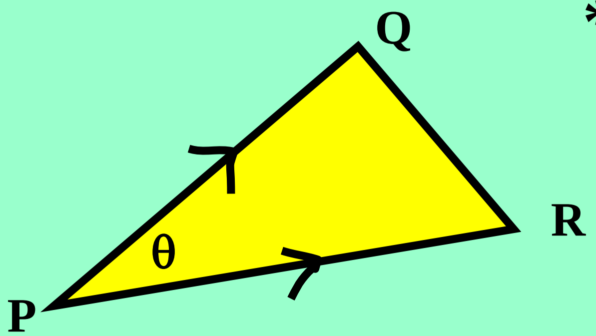
Heinemann, p.253, Ex 13Q, Q1 (d) (e) & (f)

This is not the end

<u>Example 2</u> P is (3,2,1) , Q is (7,0,5) & R is (11,2,-5).

NAB

Find the size of $Q\hat{P}R$.



****

$\overrightarrow{PQ} = \underline{q} - \underline{p} = \begin{pmatrix} 7 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$

$|PQ| = \sqrt{(4^2 + (-2)^2 + 4^2)}$

$= \sqrt{36}$

$= \underline{6}$

$\overrightarrow{PR} = \underline{r} - \underline{p} = \begin{pmatrix} 11 \\ 2 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ -6 \end{pmatrix}$

$|PR| = \sqrt{(8^2 + 0^2 + (-6)^2)}$

$= \sqrt{100}$

$= \underline{10}$

$$\vec{PR}.\vec{PQ} = (4 \text{ X } 8) + (-2 \text{ X } 0) + (4 \text{ X } (-6)) = 32 + 0 - 24 = \underline{8}$$

$$\cos\theta = \frac{\vec{PR}.\vec{PQ}}{|PQ||PR|} = \frac{8}{6 \text{ X } 10} = {}^{2}/_{15}$$

$$\theta = \cos^{-1}({}^{2}/_{15}) = \boxed{82.3°}$$

Heinemann, p.253, Ex 13Q, Q1 (a) (b) & (c) then Q2 – 6