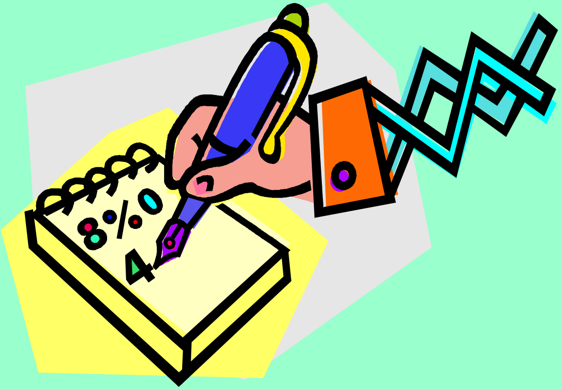
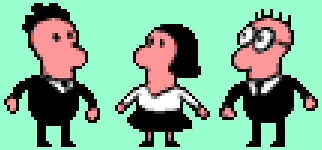
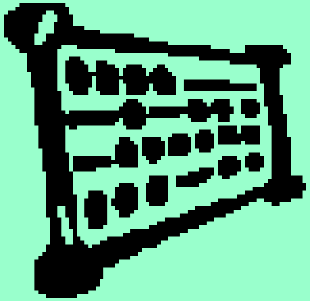
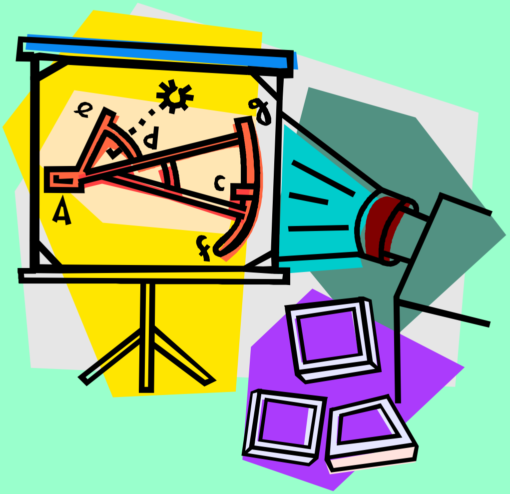




1. Vocabulary of polynomials





What is a polynomial?

A polynomial is essentially an expression which has more than one variable.

For our purposes this means an expression which has x's or y's (or any other letter) with different powers:

Examples

$3x^4 – 5x^3 + 6x^2 – 7x - 4$	**Polynomial in x of degree 4.**
$7m^8 – 5m^5 – 9m^2 + 2$	**Polynomial in m of degree 8.**
$w^{13} – 6$	**Polynomial in w of degree 13.**

NB: It is not essential to have all the powers from the highest down, however powers should be in descending order.

Disguised Polynomials

$$(x + 3)(x - 5)(x + 5) \quad = (x + 3)(x^2 - 25) \;=\; x^3 + 3x^2 - 25x - 75$$

So this is a polynomial in x of degree 3.

Coefficients

In the polynomial $3x^4 - 5x^3 + 6x^2 - 7x - 4$ **we say that**

the coefficient of x^4 **is** 3

the coefficient of x^3 **is** -5

the coefficient of x^2 **is** 6

the coefficient of x **is** -7

and the coefficient of x^0 **is** -4 ← the constant

In $w^{13} - 6$ **, the coefficients of** $w^{12}, w^{11}, \ldots w^2, w$ **are all zero.**

Evaluating Polynomials

Suppose that f(x) = $2x^3 - 4x^2 + 5x - 9$ and we wish to find the find the value of the function when x = 2

Substitution Method

$$f(2) = 2\times(2)^3 - 4\times(2)^2 + 5\times(2) - 9$$

f(2) = (2 x 2 X 2 X 2) – (4 X 2 X 2) + (5 X 2) - 9

= 16 – 16 +10 - 9

= 1

NB: this requires 9 calculations.

Nested or Synthetic Method

This involves using the coefficients and requires fewer calculations so is more efficient.

$$g(x) = 2x^3 - 4x^2 + 5x - 9$$

Coefficients are 2, -4, 5, -9

	x^3	x^2	x	x^0
g(2) → 2	2	-4	5	-9
		4	0	10
	2	0	5	**1**

g(2) = 1

This requires only 6 calculations so is $^1/_3$ more efficient.

Example

If $f(x) = 2x^3 - 8x$ find f (2)

the coefficients are 2 0 -8 0

and so f(2) → 2

	x^3	x^2	x	x^0
2	2	0	-8	0
		4	8	0
	2	4	0	**0**

If f(x) = 0 then x = 2 must be a root

Heinemann, EX 7A ,
Ex 7B,