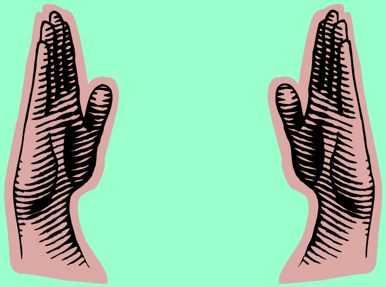
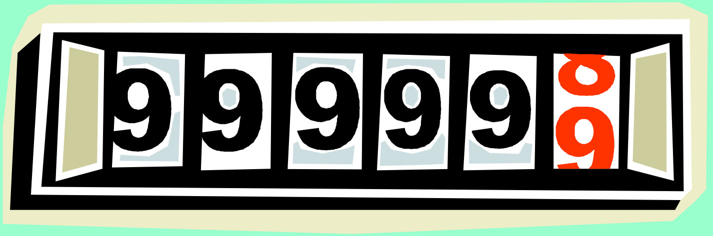




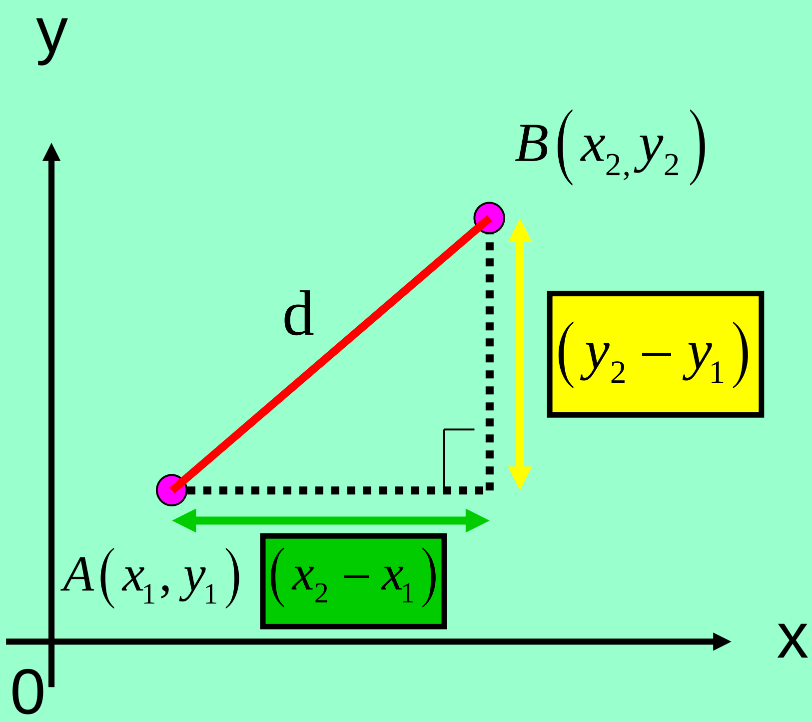
1. The Distance Formula

The Distance Formula

We have two points on the coordinate grid, A and B.

We want to find the distance, d, between these two points. This is represented by the red line.

By adding the dotted lines we can form a right-angled triangle. This allows us to use Pythagoras.



However, before we can use Pythagoras we need to find the lengths of the green line and the yellow line.

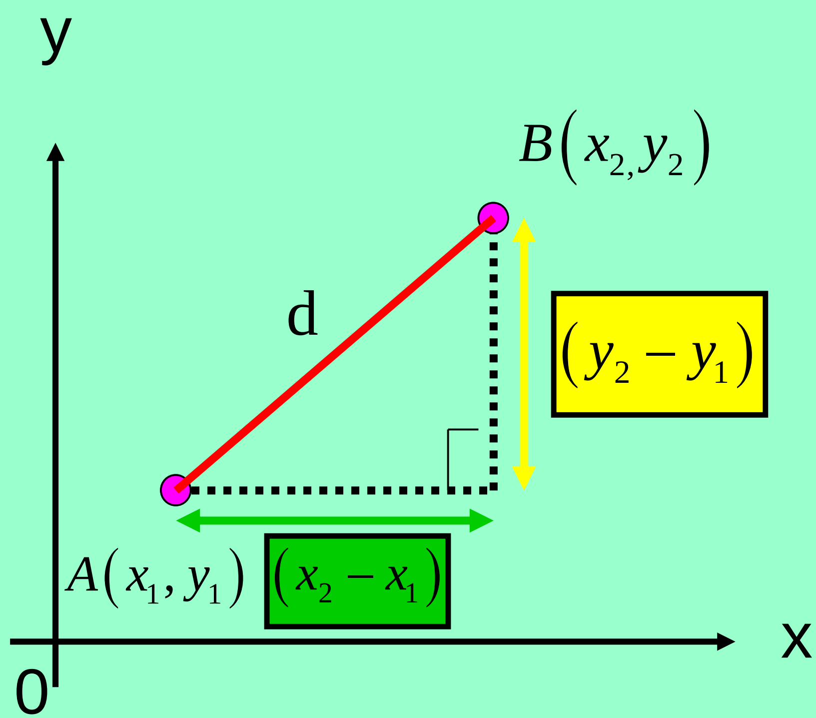
Copy the following

The Distance Formula

By Pythagoras:

$$d^2 = \left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2$$

$$\Rightarrow d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$



Example 1

Calculate the distance between the points C (-1, 3) and D (2, 7)

$\left(x_1, y_1\right)$ $\left(x_{2,} y_2\right)$

Solution:

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$d = \sqrt{(2-(-1))^2 + (7-3)^2}$$

Watch double negative !

$$d = \sqrt{3^2 + 4^2} \quad = \sqrt{25} \quad = 5 \quad \text{units}$$

Example 2

If the distance between the two points (3 , 4) and (7 , k) is $\sqrt{65}$ find the value of k.

(x_1, y_1) (x_2, y_2)

Solution:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore \sqrt{65} = \sqrt{(7-3)^2 + (k-4)^2}$$

Square both sides to eliminate square root

$$65 = (7-3)^2 + (k-4)^2$$

$$65 = 16 + (k-4)^2$$

$$49 = (k-4)^2$$

Square roots produce BOTH +VE and –VE answers

$$(k-4) = \sqrt{49} = \pm 7$$

$$k = \pm 7 + 4$$

7 + 4 or -7 + 4

k = 11 or k = -3

Example 3

P is the point (-6 , -4), Q is (2 , 2) and R is (-2 , 4).
Prove that the triangle is right-angled.

Solution:

$$PQ = \sqrt{(2-(-6))^2 + (2-(-4))^2}$$

$$PQ = \sqrt{8^2 + 6^2}$$

$$PQ = \sqrt{100}$$

$$QR = \sqrt{(-2-2)^2 + (4-2)^2}$$

$$QR = \sqrt{(-4)^2 + 2^2}$$

$$QR = \sqrt{20}$$

$$PR = \sqrt{(-2-(-6))^2 + (4-(-4))^2}$$

$$PR = \sqrt{4^2 + 8^2}$$

$$PR = \sqrt{80}$$

Solution (continued):

$$\sqrt{a} \times \sqrt{a} = a$$

$$PQ = \sqrt{100} \qquad PR = \sqrt{80} \qquad QR = \sqrt{20}$$

$$PQ^2 = 100 \qquad PR^2 = 80 \qquad QR^2 = 20$$

Since $PQ^2 = PR^2 + QR^2$ the triangle PQR is right-angled, by the Converse of Pythagoras.

Heinemann , p.211, EX 12B (ALL)