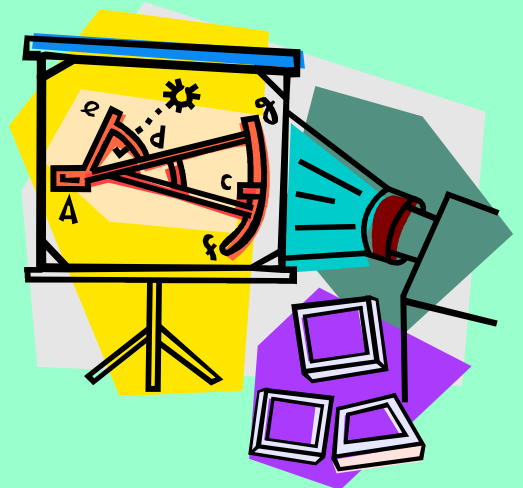
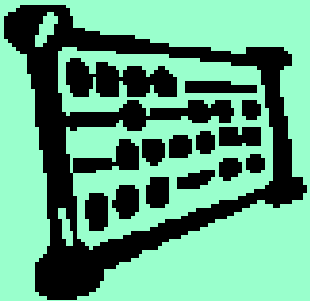


1. Introducing Wave Function

$$a \cos x + b \sin x = k \cos(x - \alpha)$$



Adding two waves

When studying fluids or sound scientists often have to analyse data in the form of what we would consider to be trigonometric waves.

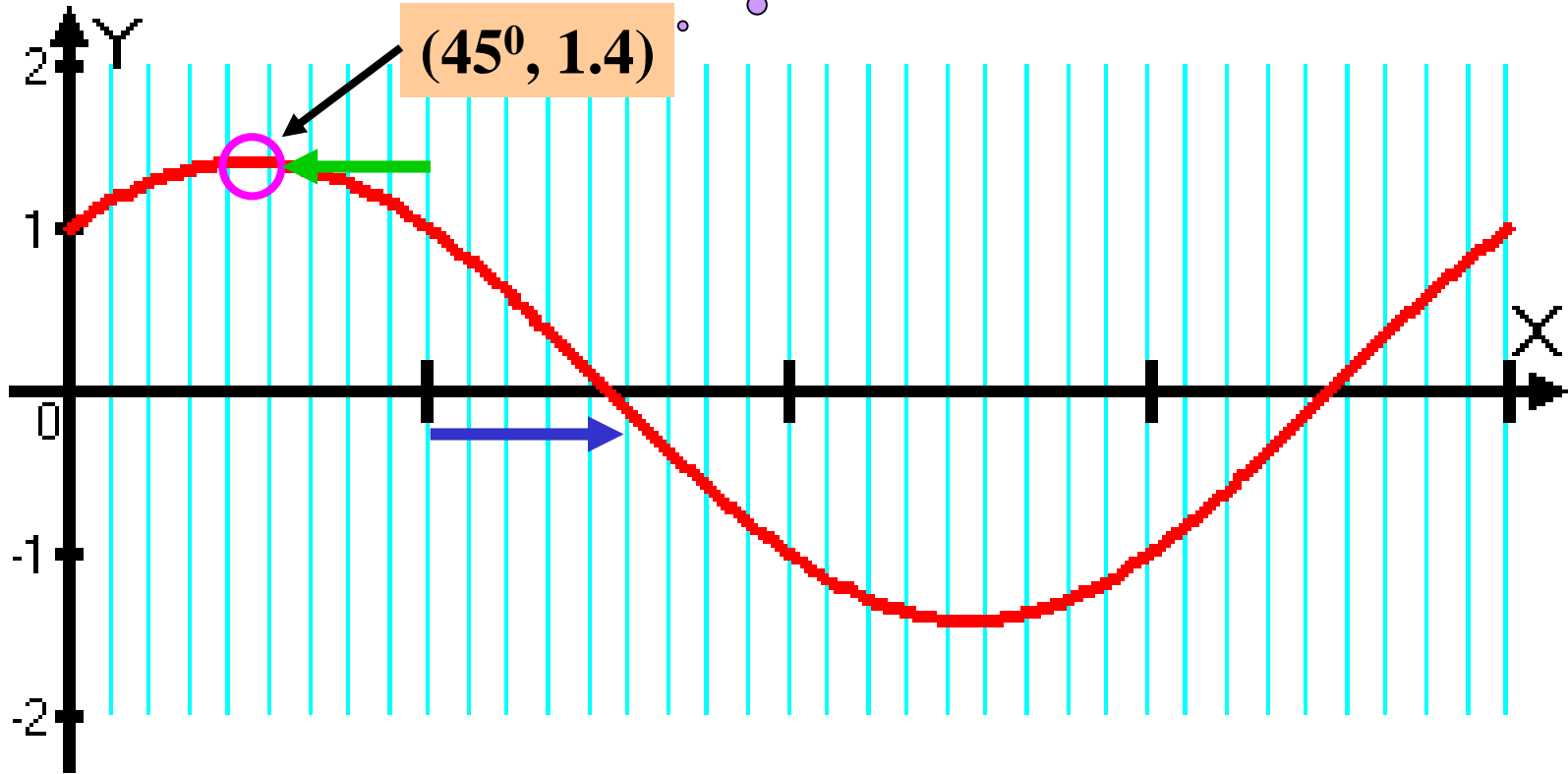
However, real-life data does not always fit nicely into a sin function wave or a cos function wave. Sometimes results can be a combination of both waves.

When finding results we usually want solutions, and we only know how to solve trig equations involving a single trig function.

So now we are going to look at how to take combined functions and write them as a single function, known as the wave function.

What is the function?

Phase angle, amplitude



Writing this function in the terms of $\sin x$ we get:

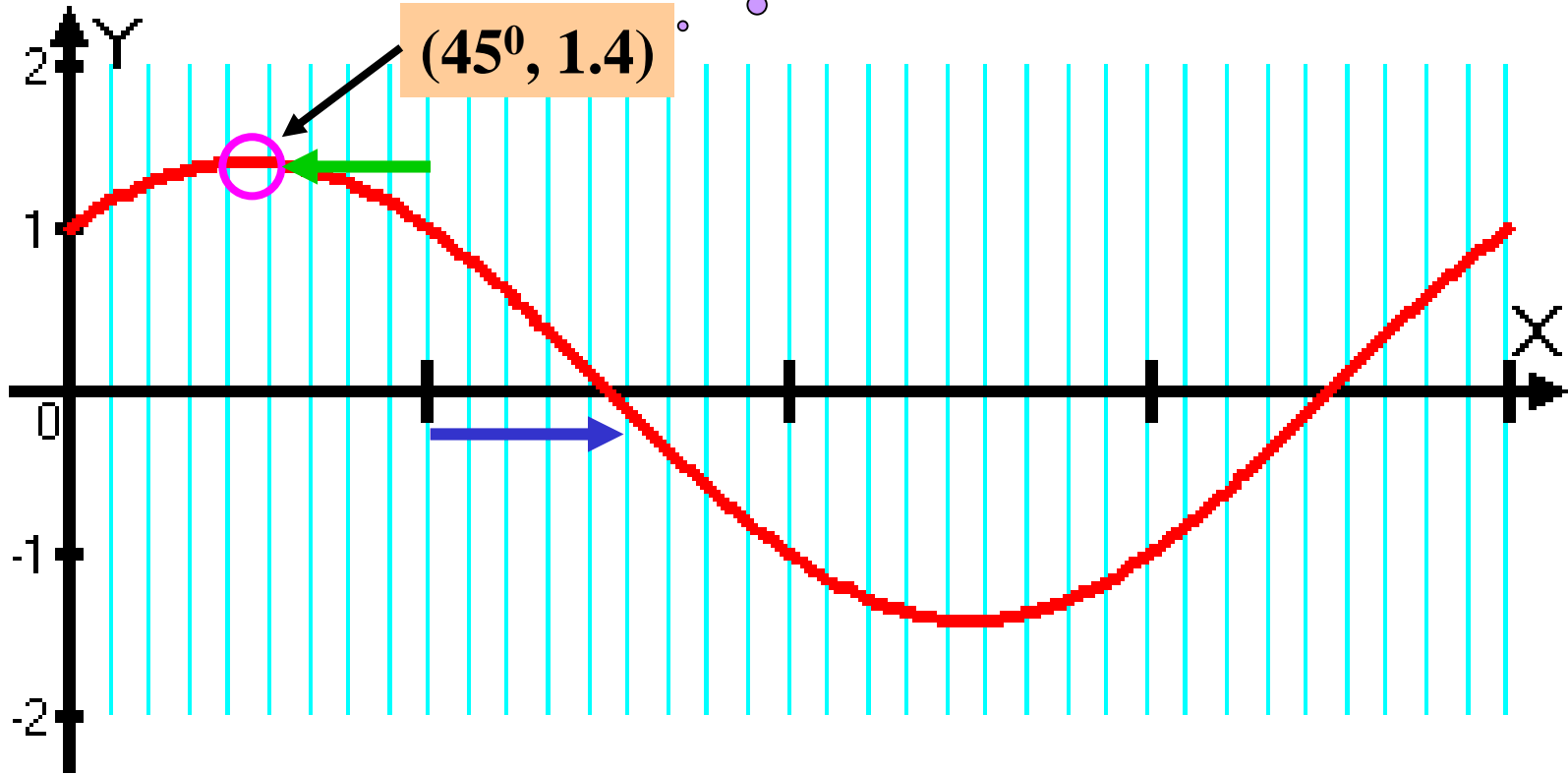
$$y = 1.4 \sin(x + 45)^\circ$$

Writing this function in the terms of $\cos x$ we get:

$$y = 1.4 \cos(x - 45)^\circ$$

What is the function?

Phase angle, amplitude



In fact this is the graph of $y = \sin x + \cos x$

and this shows that a function involving both
sin and cos can be written in terms of either.

$$y = 1.4 \sin(x + 45)^\circ$$

$$y = 1.4 \cos(x - 45)^\circ$$

The wave function

Copy the following:

$$a \cos x + b \sin x = k \cos(x - \alpha)$$

where $k = \sqrt{a^2 + b^2}$ and $\tan \alpha = \frac{b}{a}$

and k is the amplitude and α is the phase angle.

Example 1

NAB

Write $3\cos x - 4\sin x$ in the form $k \cos(x - a)$, $k > 0$, $0 \leq a < 360$

Solution:

1. Use addition formulae to expand brackets of desired "form".
2. Compare coefficients of $\sin x$ and $\cos x$ for this expansion and original.
3. Use these facts to find which quadrant a will lie in.

$$k \cos(x - a)$$

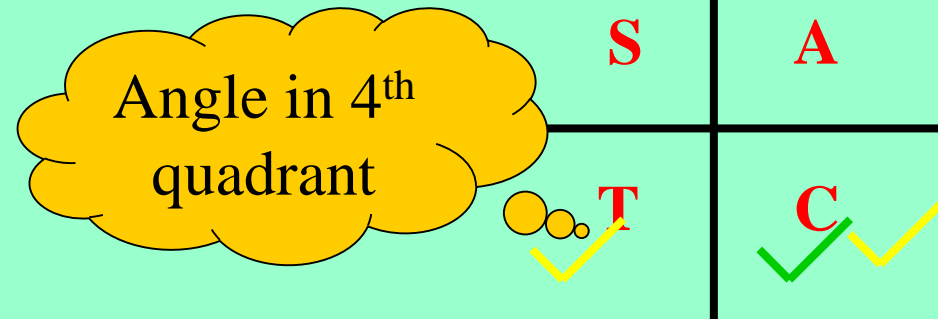
$$= k(\cos x \cos a + \sin x \sin a)$$

$$= k \cos x \cos a + k \sin x \sin a$$

$$3 \cos x - 4 \sin x$$

$$\Rightarrow k \sin a = -4$$

$$\Rightarrow k \cos a = 3$$



Example 1

NAB

Write $3\cos x - 4\sin x$ in the form $k \cos(x - a)^\circ$, $k > 0$, $0 \leq a^\circ < 360$

Solution:

4. Find k

Can leave as a surd

5. Find a

$$k = \sqrt{a^2 + b^2}$$

$$= \sqrt{3^2 + (-4)^2}$$

$$= \sqrt{25} = 5$$

Angle in 4th
quadrant

$$\tan a = \frac{\sin a}{\cos a} = \frac{-4}{3}$$

$$\Rightarrow a = -\tan^{-1}\left(\frac{4}{3}\right)$$

$$\Rightarrow a = 360^\circ - 53.1^\circ = 306.9^\circ$$

Example 1

NAB

Write $3\cos x - 4\sin x$ in the form $k\cos(x - a)^\circ$, $k > 0$, $0 \leq a^\circ < 360$

Solution:

6. Write expression in desired form.

$$k = 5 \quad a = 306.9^\circ$$

$$3\cos x - 4\sin x = 5\cos(x - 306.9)^\circ$$

Heinemann, p.304, EX 16C, Q1, 2 & 3