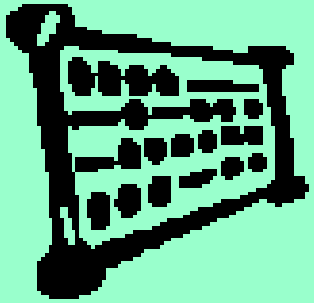
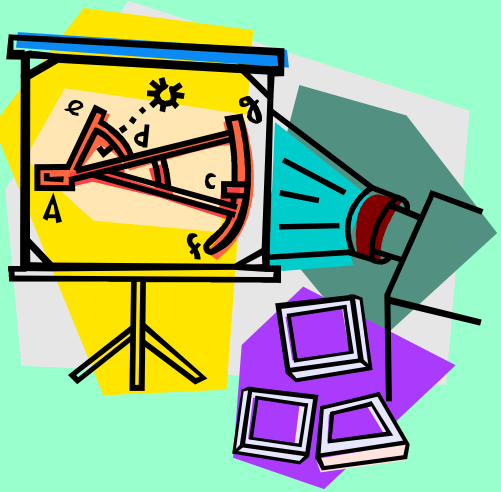
1. Exponential growth and decay





Exponential Functions

An exponential function is any function in the form:

$$y = a^x$$

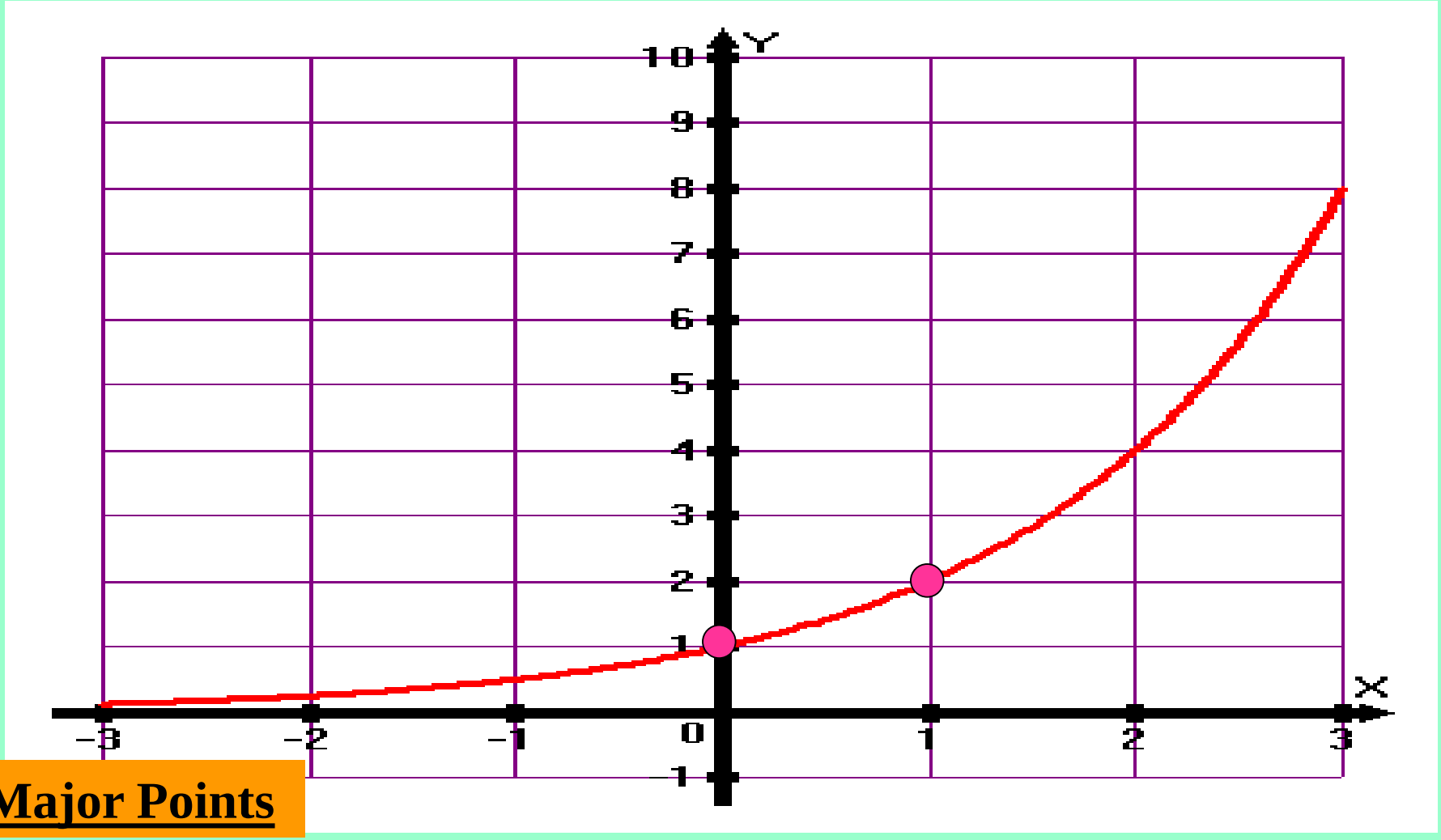
Exponent (power)

Base (must be positive)

Consider $f(x) = 2^x$

x	-3	-2	-1	0	1	2	3
y	$^1/_8$	¼	½	1	2	4	8

These points lead to the following graph:



Major Points

(i) $y = 2^x$ passes through the points (0,1) & (1,2) .

(ii) As $x \to \infty$ $y \to \infty$ however as $x \to -\infty$ $y \to 0$.

(iii) The graph shows a GROWTH function.

Example 1

NAB

If $y = 6^{3.2}$ find an approximation for y.

Solution:

Use these keys on your calculator: x^y or y^x or $\wedge$

Press **6** x^y **3.2**

$$6^{3.2} \approx 309.1$$

Heinemann, p.280, EX 15A, Q1

This is not the end

Example 2

Solve for x: $(a)\ 7^x = 117649$ $(b)\ 6^x = 42$

An exact solution for $a^x = n$ is:

$$x = \frac{\log(n)}{\log(a)}$$

Solution:

1. Make a guess $(a)\ 7^x = 117649$

Too big!! $\longrightarrow$ $7^{10} = 282475249$ 7 x^y 10

Too small!! $\longrightarrow$ $7^5 = 16807$ 7 x^y 5

Too big!! $\longrightarrow$ $7^8 = 5764801$

Bingo!! $\longrightarrow$ $7^6 = 117649$ $\Rightarrow x = 6$

Example 2

Solve for x: $(a)\ 7^x = 117649$ $(b)\ 6^x = 42$

Solution:

a **n**

$(b)\ 6^x = 42$

An exact solution for $a^x = n$ is:

$$x = \frac{\log(n)}{\log(a)}$$

$$x = \frac{\log(42)}{\log(6)}$$

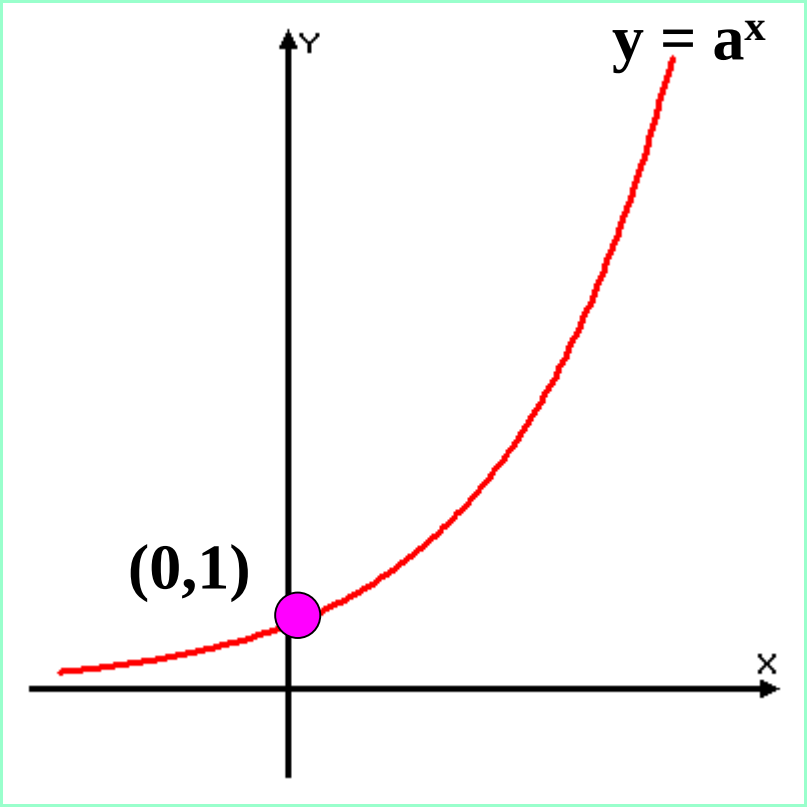
$$x = 2.08603....$$

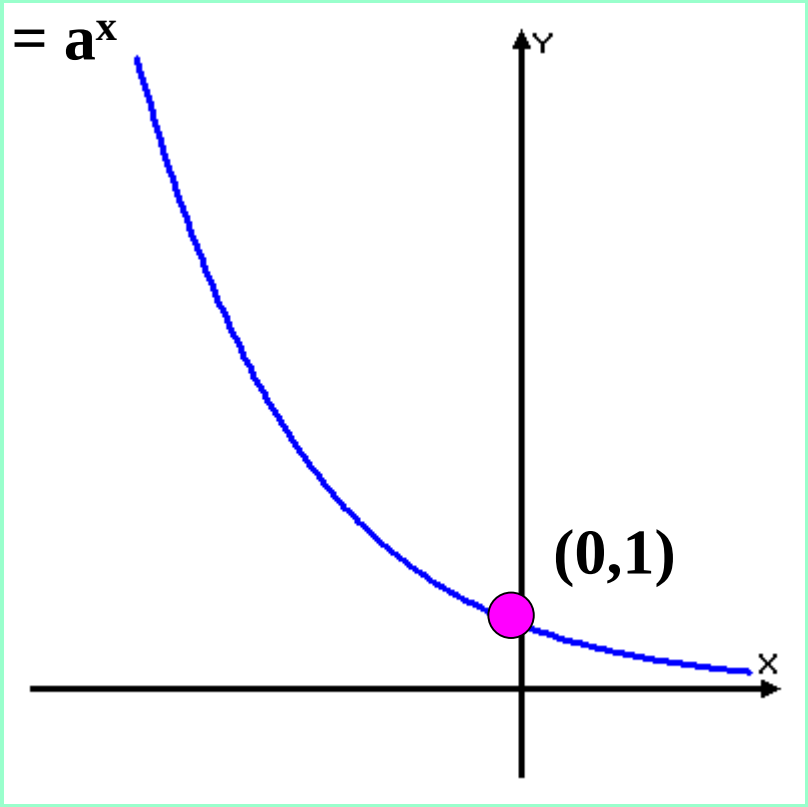
$$\Rightarrow x \approx 2 \cdot 1$$

Heinemann, p.280, EX 15A, Q3

This is not the end

Exponential Growth and Decay



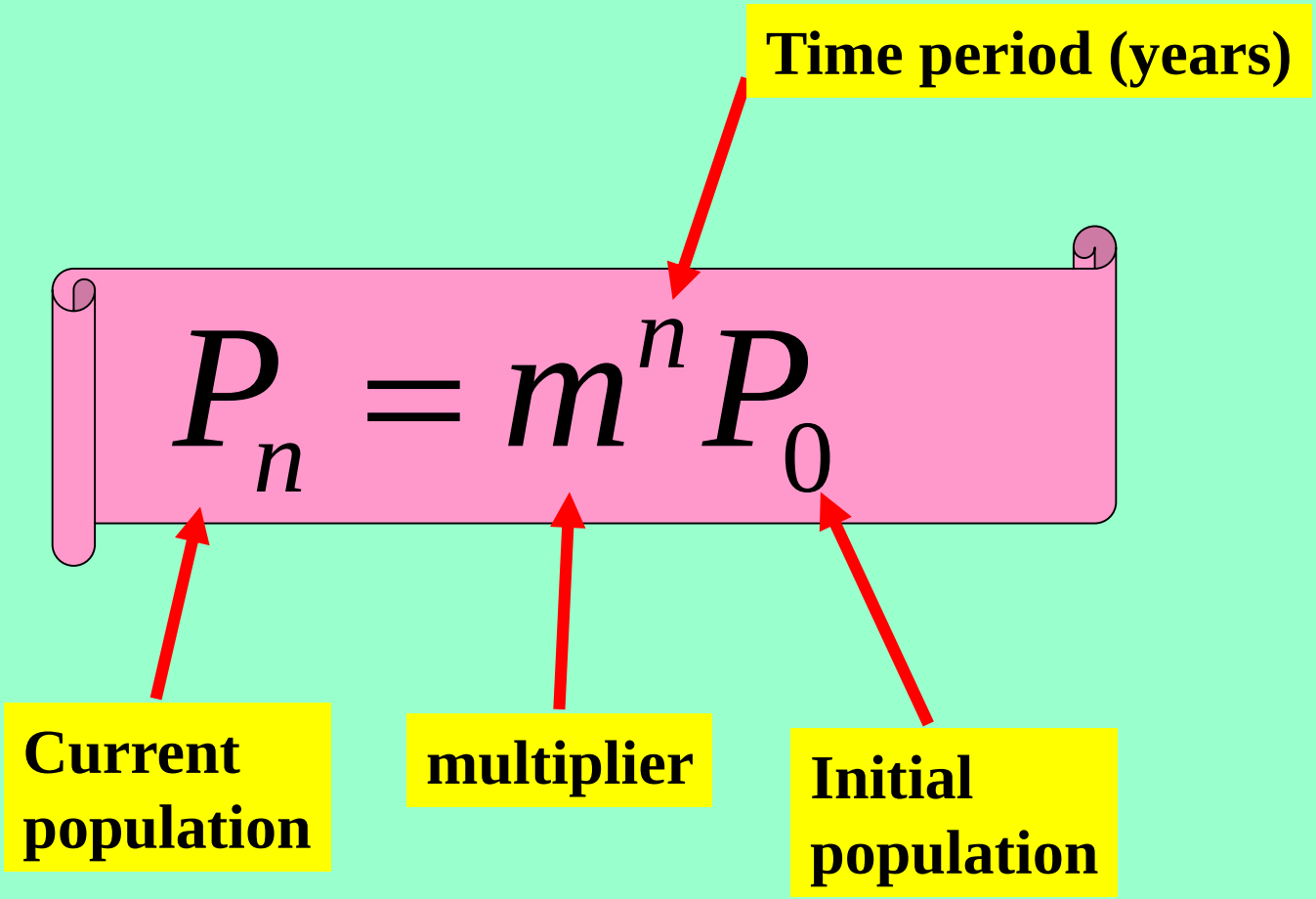


If base is bigger than 1 we have <u>exponential growth</u>.

If base is between 0 and 1 we have <u>exponential decay</u>.

If base is 1 we get straight line y = 1

A useful formula



Example 1

$$P_n = m^n P_0$$

The population of developing nation is increasing at a rate of 15% per annum. How long will it take for the population to double?

Solution:

$P_n = double = 2$

$m = 100\% + 15\% = 1 \cdot 15$

$n = ????$

$P_0 = 1$

$$P_n = m^n P_0$$

$$\Rightarrow 2 = 1.15^n \times 1$$

$$\Rightarrow 1.15^n = 2$$

$$n = \frac{\log(2)}{\log(1.15)}$$

$$n = 4.9594...$$

So population doubles within 5 years

Heinemann, p.283, EX 15C, Q1

This is not the end

Example 2

$$P_n = m^n P_0$$

The population of Scotland is decreasing at a rate of 2% per annum. How long will it take for the population to:

(a) drop from 5million to 4.4 million?

(b) halve ?

Solution to (a):

$$P_n = m^n P_0$$

$P_n = 4 \cdot 4 \ \ million$

$m = 100\% - 2\% = 0 \cdot 98$

$\Rightarrow 4 \cdot 4 = 0.98^n \times 5$

$n = ????$

$\Rightarrow 0.98^n = \frac{4 \cdot 4}{5} = 0 \cdot 88$

$P_0 = 5 \ \ million$

$$n = \frac{\log(0.88)}{\log(0.98)}$$

So population drops to 4.4m within 7 years

$n = 6.3275...$

Example 2

$$P_n = m^n P_0$$

The population of Scotland is decreasing at a rate of 2% per annum. How long will it take for the population to:

(a) drop from 5million to 4.4 million?

(b) halve ?

Solution to (b):

$$P_n = 1$$

$$m = 100\% - 2\% = 0 \cdot 98$$

$$n = ????$$

$$P_0 = 2$$

$$P_n = m^n P_0$$

$$1 = 0.98^n \times 2$$

$$\Rightarrow 0.98^n = \tfrac{1}{2} = 0 \cdot 50$$

$$n = \frac{\log(0.50)}{\log(0.98)}$$

$$n = 34.3096...$$

So population halves within 35 years

Heinemann, p.283, EX 15C, Q2, 3 & 7