

1.

Equation of Circle - Centre the Origin



$$x^2 + y^2 = r^2$$



Equation of a circle with centre the origin

Take any point on the circumference of a circle which has the origin as its centre.

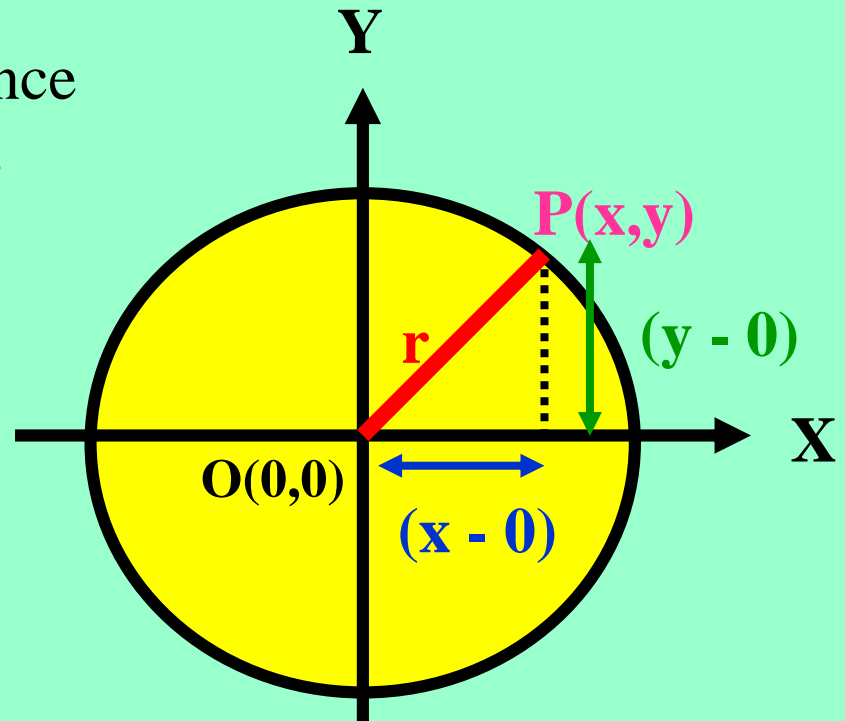
Applying the distance formula we get:

$$r^2 = (x - 0)^2 + (y - 0)^2$$

$$\text{So } r^2 = x^2 + y^2$$

This gives us the equation for a circle with centre, O :

$$x^2 + y^2 = r^2$$



Equation of a circle with centre the origin

Copy the following:

The equation of any circle with centre O and radius r is:

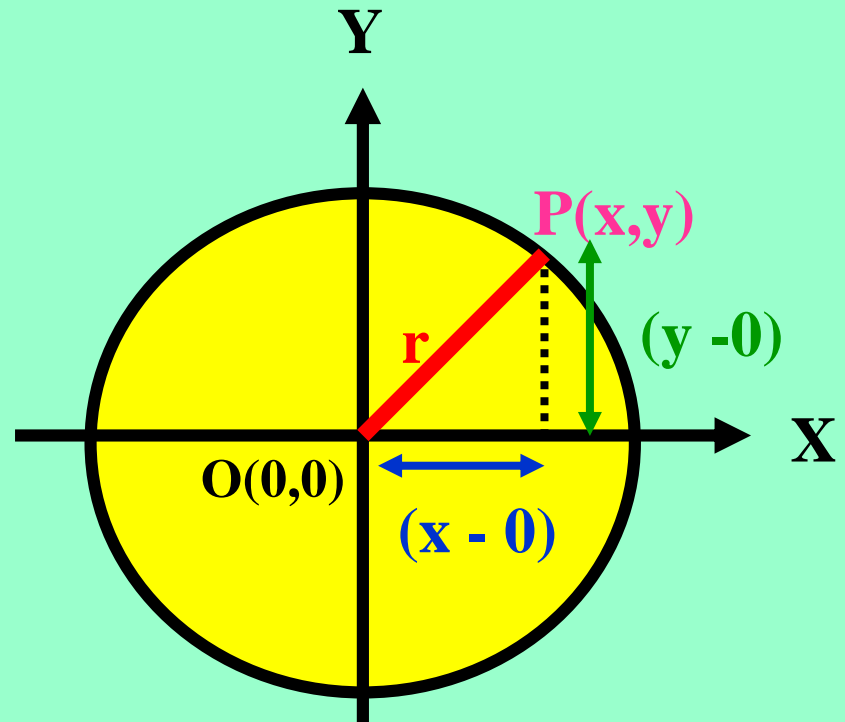
$$x^2 + y^2 = r^2$$

If a point (x,y) lies:

INSIDE the circle then $x^2 + y^2 < r^2$

OUTSIDE the circle then $x^2 + y^2 > r^2$

ON THE CIRCUMFERENCE then $x^2 + y^2 = r^2$



Example 1

(a) These circles have centre O. Write down their equations:

(i) if $r = 3$

$$x^2 + y^2 = 3^2$$

$$x^2 + y^2 = 9$$

(ii) if the circle passes through (8,15)

For all points : $x^2 + y^2 = r^2$

For (8,15)

$$(8)^2 + (15)^2 = r^2$$

$$64 + 225 = r^2$$

$$r^2 = 289$$

Equation for
all points:

$$x^2 + y^2 = 289$$

Example 1

(b) Find the radius of the circle with equation:

(i) $x^2 + y^2 = 16$

$$r^2 = 16$$

$$r = \sqrt{16}$$

$$r = 4$$

(ii) $x^2 + y^2 = 50$

$$r^2 = 50$$

$$r = \sqrt{50}$$

$$r = \sqrt{25 \times 2}$$

$$r = 5\sqrt{2}$$

(iii) $x^2 + y^2 = 23$

$$r^2 = 23$$

$$r = \sqrt{23}$$

Example 2

Does the point (9,2) lie on the circumference of the circle with equation $x^2 + y^2 = 90$?

Solution:

For (9,2): $(9)^2 + (2)^2 = 85$

As 85 is less than 90, the point does not lie on the circumference it lies inside the circle.

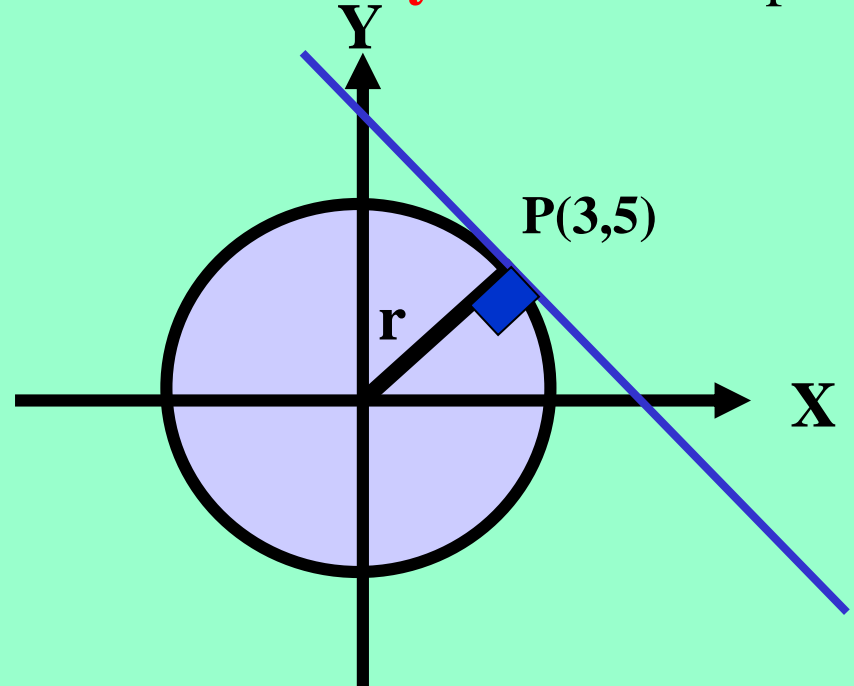
Example 3

NAB

Find the equation of the tangent to the circle $x^2 + y^2 = 34$ at the point $(3,5)$

Solution:

Tangent =
straight line so
need **m** and
point



Gradient between
 $(0,0)$ and $(3,5)$:

$$m = \frac{5-0}{3-0} = \frac{5}{3}$$

Gradient
of tangent:

$$m_{\text{tan}} = -\frac{3}{5}$$

Equation of tangent:

$$y - 5 = -\frac{3}{5}(x - 3) \quad \times 5$$

$$5y - 25 = -3(x - 3)$$

$$5y - 25 = -3x + 9$$

$$5y + 3x = 34$$

Heinemann,
p.207, EX 12D, Q1, 2, 3 & 10 (a) to (d)