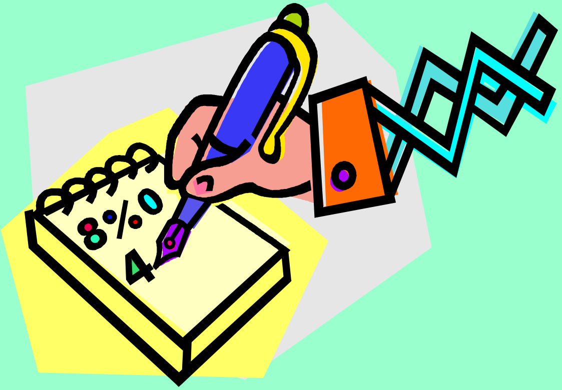
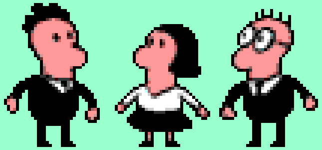
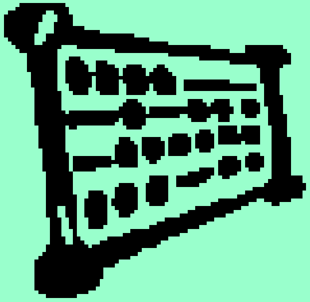
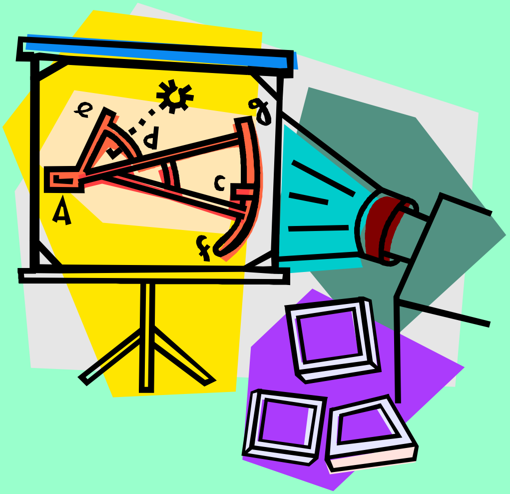




1. Anti-differentiation

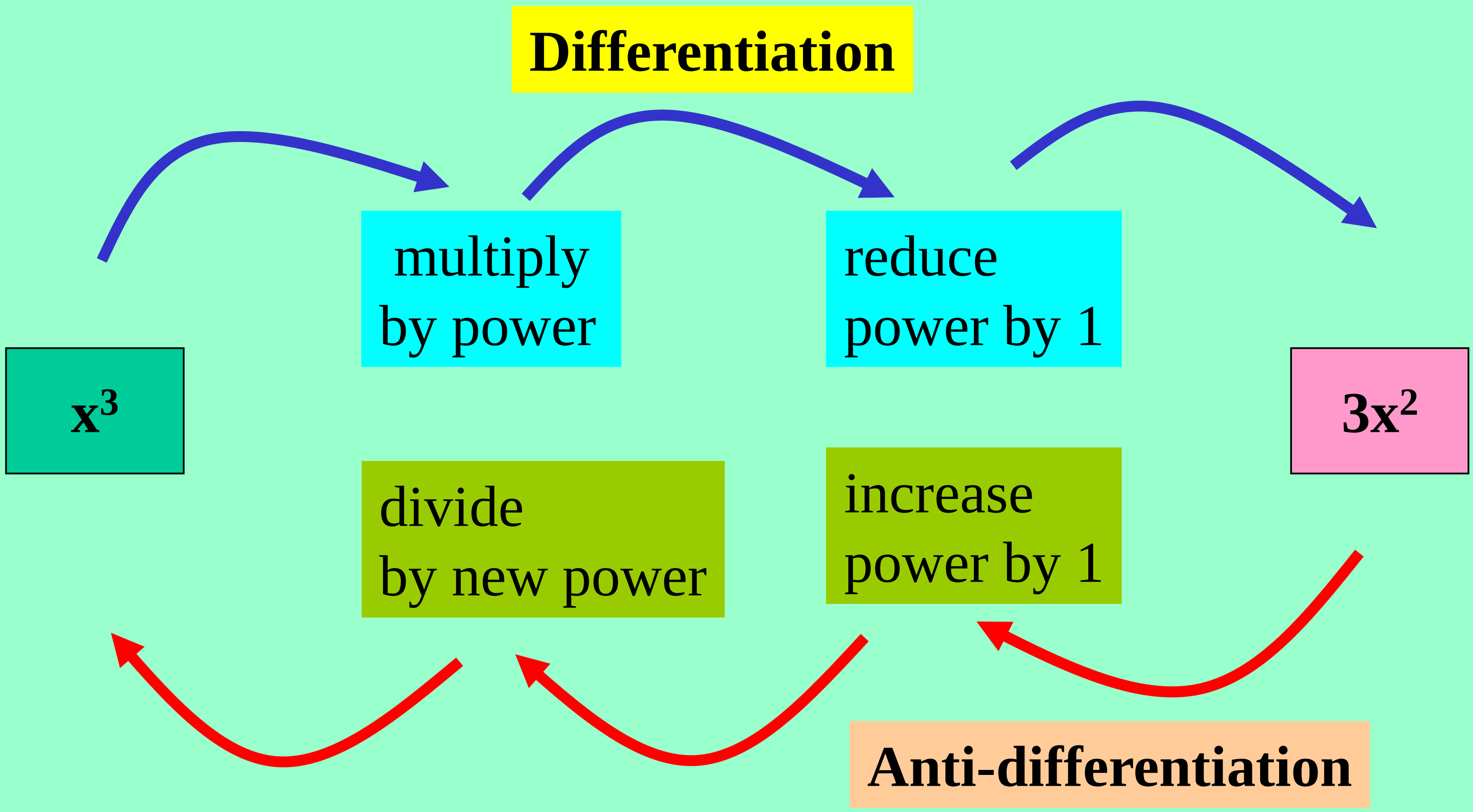




What is anti-differentiation?

Just as many functions and operations have opposites or inverses which "undo" them , so too does differentiation.

Take the function $f(x) = x^3$



What is anti-differentiation?

The process of finding anti-derivatives is called **integration**.

However, there is a slight complication when we integrate.

Differentiate: (i) x^4 (ii) $x^4 + 8$ (iii) $x^4 - 4$

$f'(x) = 4x^3$ $f'(x) = 4x^3$ $f'(x) = 4x^3$

All of these curves have the same derivative. Whilst they are all part of the same "family" of curves, they are not all the same curve.

Therefore, to overcome this problem, until such times as we know a point on the curve, we add a constant (C) when we integrate.

Integration

Copy the following:

- the opposite of differentiation

The symbol for integration is $\int$ and there are two forms:

Indefinite Integrals	Definite Integrals
$\int f(x)dx$	$\int_a^b f(x)dx$
No limits	a and b are limits
Answers algebraic with constant	Answers numerical- no constant

$$\int f(x)dx = \int F(x) + c$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

Basic Rules

1. $$\int x^n dx = \frac{x^{(n+1)}}{(n+1)} + C \qquad n \neq -1$$

2. $$\int ax^n dx = \frac{ax^{(n+1)}}{(n+1)} + C \qquad n \neq -1$$

3. $$\int f(x) + g(x) dx = \int f(x) dx + \int g(x) dx$$

Differentiate :
Decrease Power

Integrate :
Increase Power

Examples of Rule 1

Find:

1. $$\int x^7 dx$$

increase power by 1

$$= \frac{x^8}{8} + C$$

divide by new power

2. $$\int \frac{1}{u^3}\, du \quad = \int u^{-3} du$$

$$= \frac{u^{-2}}{-2} + C \quad = \frac{1}{-2u^2} + C \quad = -\frac{1}{2u^2} + C$$

3. $$\int p^{\frac{2}{3}} dp \quad = \frac{p^{\frac{5}{3}}}{\frac{5}{3}} + C \quad = \frac{3p^{\frac{5}{3}}}{5} + C$$

Heinemann, p.164, EX 9G,
Q1(a) (b) (c) (g) (h) (i)