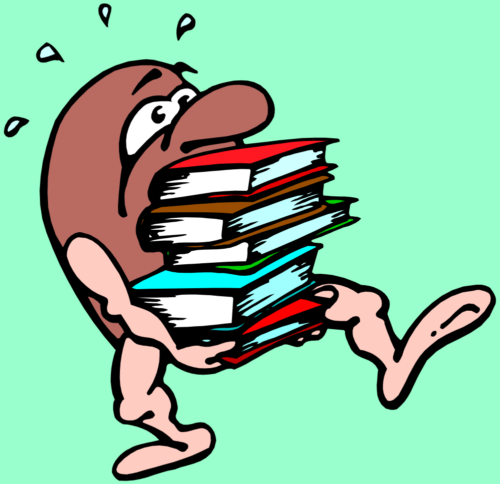
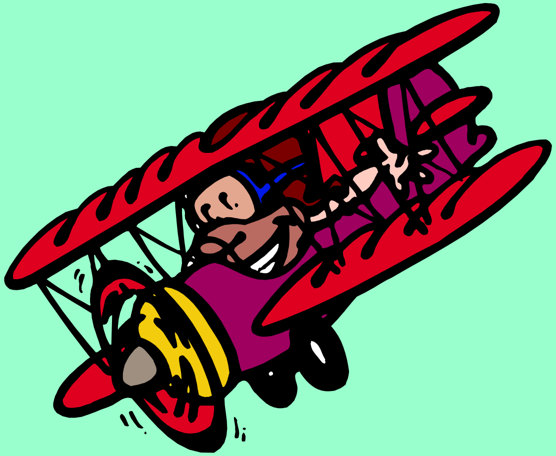
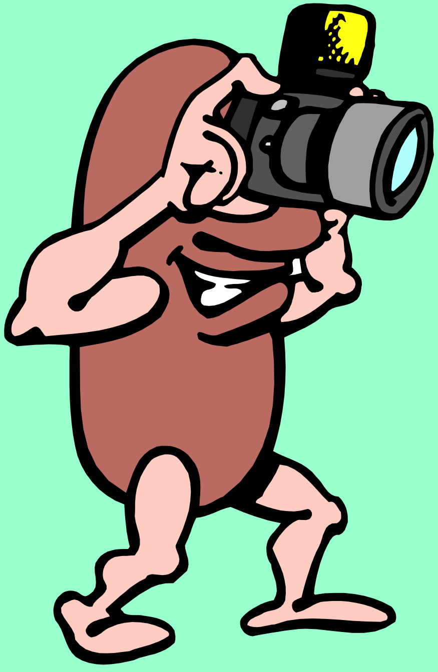




1.

Addition Formulae



ADDITION FORMULAE

Copy:

The following relationships are always true for two angles A and B.

(1)(a) sin(A + B) = sinAcosB + cosAsinB

(1)(b) sin(A - B) = sinAcosB - cosAsinB

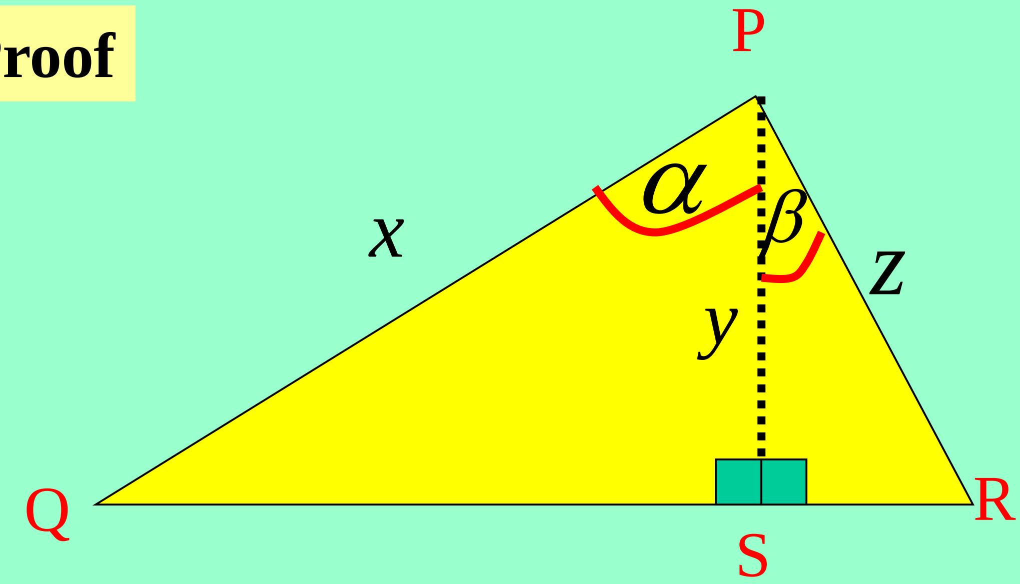
(2)(a) cos(A + B) = cosAcosB - sinAsinB

(2)(b) cos(A - B) = cosAcosB + sinAsinB

Supplied on the formula sheet !!

Quite tricky to prove but some of following examples should show that they do work!!

Proof



$$\cos\alpha = \frac{y}{x}$$

$$\cos\beta = \frac{y}{z}$$

$$Area\Delta PQR = Area\Delta PQS + Area\Delta PRS$$

$$A = \tfrac{1}{2}ab\sin C$$

$$\tfrac{1}{2}xz\sin(\alpha+\beta) = \tfrac{1}{2}xy\sin\alpha + \tfrac{1}{2}yz\sin\beta$$

$$xz\sin(\alpha+\beta) = xy\sin\alpha + yz\sin\beta$$

$$\sin(\alpha+\beta) = \frac{xy}{xz}\sin\alpha + \frac{yz}{xz}\sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

Example 1

(2)(a) $\cos(A + B) = \cos A\cos B - \sin A\sin B$

Expand:

(a) $\cos (P + Q)$ ← Rule 2(a)

$= \cos P \cos Q - \sin P \sin Q$

(b) $\sin (X - Y)$ ← Rule 1(b)

$= \sin X \cos Y - \cos X \sin Y$

(1)(b) $\sin(A - B) = \sin A\cos B - \cos A\sin B$

Example 2

Expand and simplify :

(a) $\cos (180 + X)^0$ ← Rule 2(a)

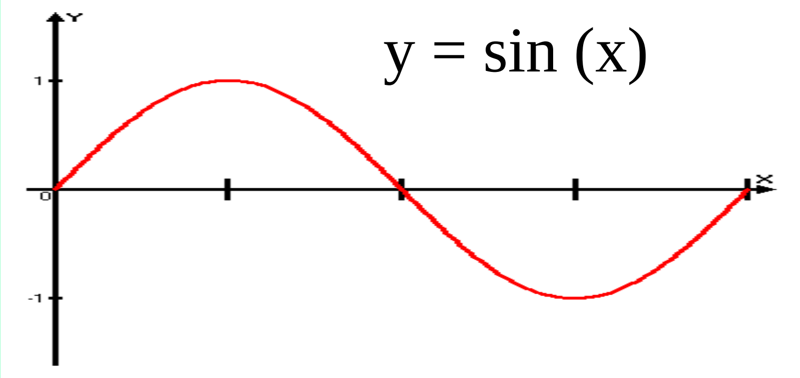
$= \cos 180^0 \cos X^0 - \sin 180^0 \sin X^0$

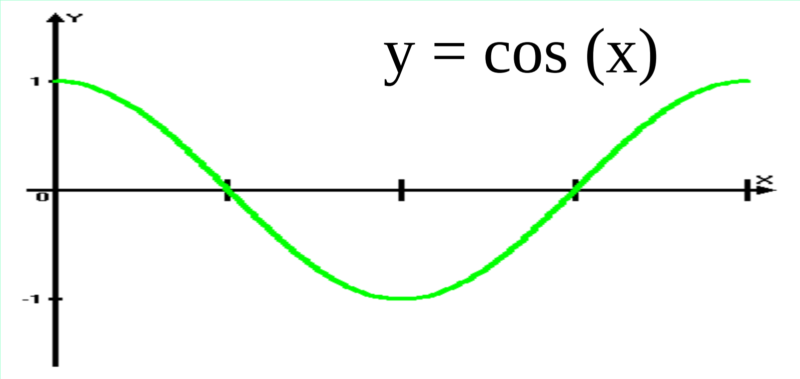
$= -\cos X^0$

(b) $\sin\left(\frac{\pi}{2} - y\right)$ ← Rule 1(b)

$= \sin 90^0 \cos y - \cos 90^0 \sin y$

$= \cos y$





Example 2

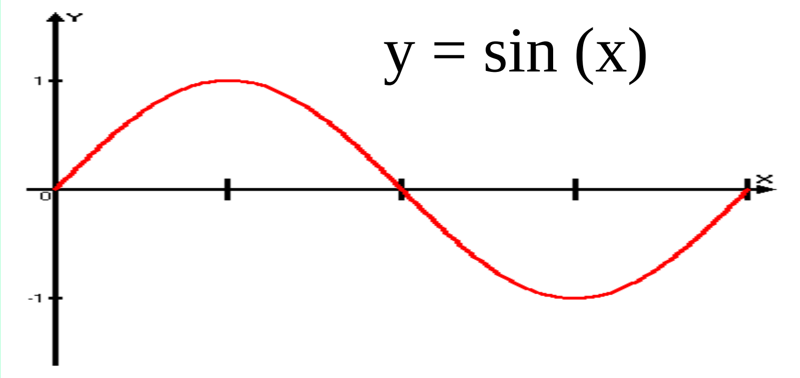
Expand and simplify :

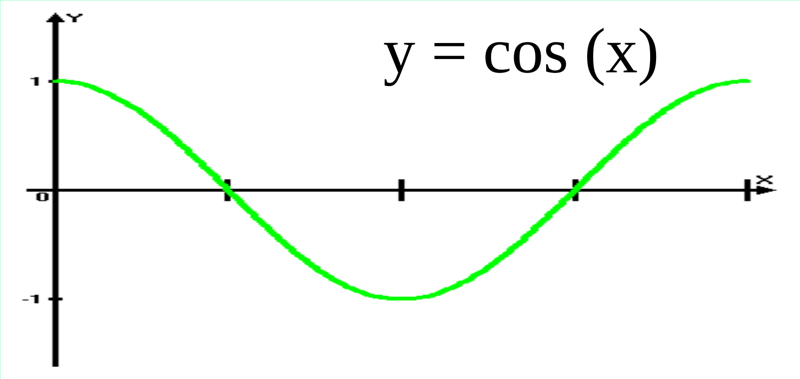
(c) $\cos(-t)^0$

$= \cos(0 - t)^0$ ← Rule 2(b)

$= \cos 0^0 \cos t^0 + \sin 0^0 \sin t^0$

$= \cos t^0$





Example 3

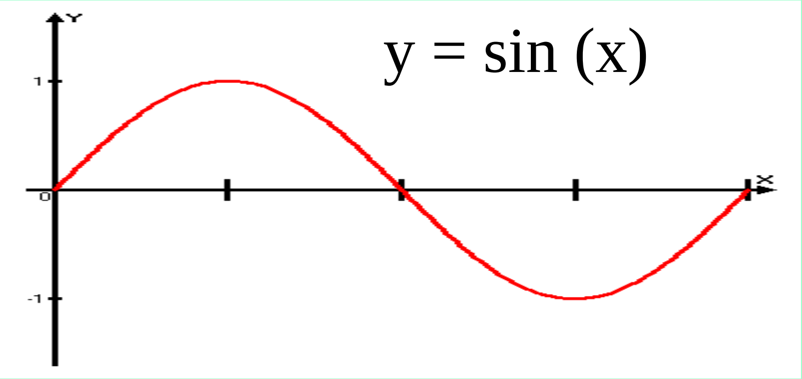
Simplify:

(a) $\sin 130^0 \cos 50^0 + \cos 130^0 \sin 50^0$ ← Rule 1(a)

$= \sin(130+50)^0$

$= \sin 180^0$

$= 0$



(b) $\cos 5y \cos y + \sin 5y \sin y$

$= \cos(5y - y)$

$= \cos 4y$

Heinemann,
p.189, EX 11B, Q1 & 4,
p. 190, EX 11C, Q1 & 5
p.191, EX 11D, Q1 & 6